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### Using Multi-constraint Integer Nonlinear Optimization Model to **Select Generator**

Rui Yan<sup>1\*</sup>, Zihan Hu<sup>1</sup>, Cheng Li<sup>1</sup>, Lijuan Li<sup>2</sup>, Rao Fu<sup>1</sup>

### **ABSTRACT**

This paper studies the power generation plan when the power is \*Correspondence to Author: cut off within a week in the factory. Considering various factors Rui Yan that affect the cost, it comprehensively analyzes the possibility of College of Hydraulic & Environmenopening the plan and the influence of three kinds of costs on the mental Engineering, China Three total cost, and establishes a multi-constraint integer nonlinear Gorges University, Yichang, optimization model. In order to minimize the total cost of a week, 443002, China. a minimum model of fixed cost, marginal cost and start-up cost was established, and a seven-day minimum cost scheme was obtained by means of dynamic programming. After changing the output power of the generator, the constraint conditions of the output power are adjusted based on the model of the above problems, and a new model of electricity demand and generation constraint Integer Nonlinear Ois obtained. In this paper, the model is extended to other problems of continuous segmentation, and the improvement can Generator. American Journal be generalized to solve such problems.

**Keywords:** Multi-constrained integer nonlinear combination optimization method; Start-up cost; Power dispatch; lingo

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<sup>&</sup>lt;sup>1</sup>College of Hydraulic & Environmental Engineering, China Three Gorges University, Yichang, 443002. China.

<sup>&</sup>lt;sup>2</sup>College of Medicine, China Three Gorges University, Yichang, 443002, China.

#### 1. Introduction

At the end of the 19th century, with the growth of electricity demand, people began to put forward the idea of establishing power production center. The development of motor manufacturing technology, the expansion of the range of electrical energy applications, the rapid growth of the demand for electricity production. Generators are widely used in industrial and agricultural production, national defense, science and technology, and in daily life, and they refer to mechanical equipment that converts other forms of energy into electrical energy, which are driven by water turbines, steam turbines, diesel engines or other power machinery, which convert seifrom, airflow, fuel combustion or atomic fission into mechanical energy to a generator and then from a generator to electrical energy. Generators come in many forms, and different types of generators have different properties.

This paper studies the issue of enabling four different types of generators to meet the daily power needs. Therefore, the establishment of a suitable model, so that production costs to a minimum, the interests of the plant is of great significance.

Existing plant due to the transformation of power facilities, in the next week will be power edging, during which time to meet the daily power demand, the plant needs to enable four different types of generators to meet the daily power demand and to minimize production costs, because of different generation performance generator costs, it can be used in different types of generator combination. All four generators have a maximum power generation capacity and a start-up cost and a fixed hourly cost when

working at their minimum power state. When connected to the grid, its output power should not be lower than its minimum output power, and if the power is higher than the minimum power, there is a cost per megawatt of excess power, i.e. marginal cost.

Minimum total cost problem: Which generators should be used separately in each session to minimize the total cost of each day, and what is the minimum total cost of the week?

Minimum total cost after reserved power: If at any one time, the working generator set must set aside 20% of the power generation capacity margin to prevent a sudden increase in electricity consumption. So which generators should be used separately for each time period to minimize the total cost of the day, at which point what is the minimum total cost of a week?

### 2. Model assumptions

- Assumption 1: When the same model is used for a continuous period of time, the default generator can be continuously turned on for each two time periods without taking into account the start-up cost.
- Assumption 2: The generator's switch is transient and the required power is achieved and remains constant for a period of time well below a time period.
- Assumption 3: All units were shut down before the first time period.
- 4) Assumption 4: Eliminates the problem of incapacity or power degradation due to a fault or other reason, i.e. all variables are human changes.
- 5) Assumption 5: Cost is only marginal cost, start-up cost and fixed cost, no other cost situation."

### 3. Symbol description

$a_{ij}$	$i \;\;  ext{Model}$ $j \;\;  ext{Output power in the first time period}$						
$x_{ij}$	i Model						
	j Number of stations to run in the first time period						
$G_{i}$	Fixed $i$ cost per hour for type of generator						
$Q_{_{i}}$	i Start-up cost per generator						
$B_i$	Marginal $i$ cost per hour for the first generator						
$T_{j}$	Total $j$ time for the first time period						
$M_{i}$	i Minimum output power for the first generator						
$N_{j}$	Power $j$ requirements for users during the first time period						
$D_{ij}$	i The quantity difference between the two time periods of the						
	type of generator						
W	Total cost of engine set per day						
i	Generator model, take 1, 2, 3, 4						
j	Time period, take 1, 2, 3, 4, 5, 6, 7						

### 4. Problem analysis

### 4.1 Analysis of minimum total cost problem

To solve minimum total cost problem, this article requires establishing a functional relationship between the user's demand for power and cost for each time of the day, and adding up the minimum cost of one week. Since the total cost is composed of start-up cost, marginal cost and fixed cost, it can be expressed as an expression of known data and three costs. It is considered when considering the start-up cost, consider

whether the unit should start separately for each time period, according to the understanding of this article assumes that the same generator can continuously work to reduce the start-up cost. Thus this article needs to relate to the relationship between each time period and the previous time period, That is, you can't just set up an equation for the total cost based on the quantity demanded in a single period. Cost-related time periods should be continuously analyzed.



Fig.1: Minimum total cost problem Flowchart

# 4.2 Analysis of minimum total cost after reserved power

The minimum total cost after reserved power is

still around the minimum cost, the main function is still related to the three costs. The problem of reserve balance is involved in the problem of requiring a 20% generating capacity of the working generator set. According to the assumptions, this article can list the relationship between the maximum output power and the reserved power, and the relationship between power generation and reserved power. It is important to note that the minimum output power

of the generator should remain the same. Cost analysis of time periods should still be continuous. Therefore, the second change is only the constraint condition, the main three function relationship does not change too much, the total relationship formula is not the same.

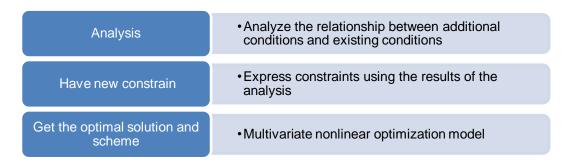


Fig.2: Minimum total cost after reserved power thought analysis

### 5. Data analysis

The question gives the demand for electricity for seven periodsof the day as shown in Table1, and the demand for electricity varies with time as shown in the daily period as shown in the following table.

Period (0-24)	0-6	6-9	9-12	12-14	14-18	18-22	22-24
Demand	11000	33000	25000	36000	25000	30000	18000

Table 1: Daily demand for electricity (KW)

The table given by the topic processes its data to obtain the relationship between the daily

demand for electricity and the image of the time period, and arrives at the following image.

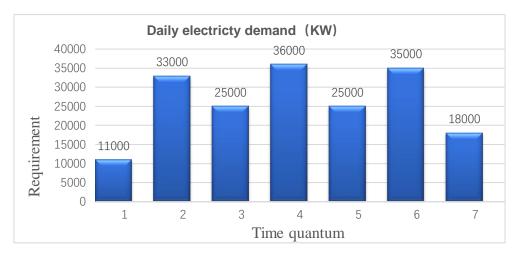


Fig.3:Daily demand for electricity (KW)

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From this diagram can be seen intuitively in different time periods of different demand for electricity, from the initial low demand to the demand for a larger stage need to increase equipment, there is an opening cost, and then a period of time electricity demand reduced, shut down excess equipment, its demand for

electricity has been changing up and down.

The following table shows the situation of generators, different generation performance of generators produce different production costs, so it is necessary to its model corresponding cost analysis to arrive at the optimal combination, so that the total cost is the lowest.

	Number available	Minimum output power (KW)	Maximum output power (KW)	Fixed cost (RMB/hour)	Marginal cost per kilowatt (RMB/hour)	Start-up costs
Model 1	10	800	1800	2200	2.7	4000
Model 2	5	1000	1500	1800	2.2	1500
Model 3	8	1200	2000	3800	1.8	2500
Model 4	4	1800	3500	4600	3.6	1000

Table 2: Generator conditions

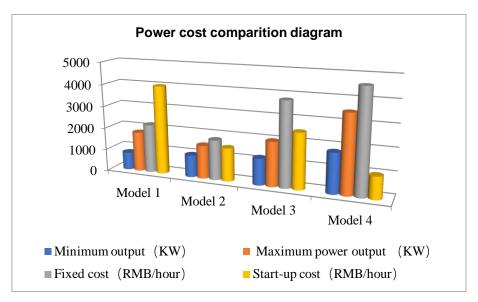


Fig.4:Acomparison of the power of different types of generators and the costs they generate

The difference between the minimum, maximum output power, fixed cost, marking cost, and startup cost of different engines can be seen intuitively from Fig.4.

- 6. The establishment and solution of minimum total cost problem
- 6.1 The establishment of the minimum total cost problemmodel
- 6.1.1 Determining the target functio

The conditions given by the title and the assumptions put forward for the model can be seen: the total cost of daily power generation in a power plant consists only of the fixed cost, marginal cost and start-up cost of the generator set, which requires the minimum solution for the cost of the plant per day, the target function is

$$\min W = G + B + Q$$

(1) The fixed cost of the four engines is

$$G = \sum_{i=1}^{4} \sum_{j=1}^{7} x_{ij} T_{j} G_{i}$$

(2) The cost of editor-in-chief when working with four types of engines is

$$B = \sum_{i=1}^{4} \sum_{j=1}^{7} (a_{ij} - M_i) x_{ij} T_j B_i$$

(3) The total start-up cost of starting the engine is

$$Q = \sum_{i=1}^{4} \sum_{i=1}^{7} \left[ D_{ij} + \left| D_{ij} \right| \right] / 2Q_{i}$$

(4) The i difference in quantity between the

first type of generator in the two-time period is

$$D_{ij} = \begin{cases} x_{ij} - x_{i(j-1)} & j = 2, 3, 4, 5, 6, 7 \\ x_{ij} - z & j = 1 \end{cases}$$

where z is for the previous day  $x_{i7}$ 

### 6.1.2 Determining constraints

The constraints given in the title are mainly related to the operating performance of the generator set and the number of units, i.e. the power limit of the generator set itself and the total number of units of each type of generator that can be called at the same time.

(1) The output power of the i generator should be less than the maximum output power of the generator, greater than the minimum output power of the generator, i.e.

$$\begin{cases} 800 \le a_{1j} \le 1800 \\ 1000 \le a_{2j} \le 1500 \\ 1200 \le a_{3j} \le 2000 \\ 1800 \le a_{4j} \le 3500 \end{cases}$$

(2) In the j period of time, the i number of type of generators called should be less than the

total number of the type of generators that can be called is not less than zero, i.e.

$$\begin{cases} 0 \le x_{1j} \le 10 \\ 0 \le x_{2j} \le 5 \\ 0 \le x_{3j} \le 8 \\ 0 \le x_{4j} \le 4 \end{cases}$$

(3) The output power per hour of the generator power per hour during that period, i.e. set shall be greater than or equal to the required

$$N_j \leq \sum_{i=1}^4 \sum_{j=1}^7 \left( a_{ij} x_{ij} \right)$$

**6.1.3 Model of minimum total cost problem** this article can get a multivariate optimization Based on the above conditions and functions, model with a problem.

$$\min W = \sum_{i=1}^{4} \sum_{j=1}^{7} x_{ij} T_{j} G_{i} + \sum_{i=1}^{4} \sum_{j=1}^{7} + \left( a_{ij} - M_{i} \right) x_{ij} T_{j} B_{i} + \sum_{i=1}^{4} \sum_{j=1}^{7} \left[ D_{ij} + \left| D_{ij} \right| \right] / 2Q_{i}$$

6.2 Solution of the minimum total cost probleme model

As shown in the table below, the following generators should be used separately in each time period to minimize the total cost per day, with a minimum total cost of 1412720 RMB on

the first day of the week , and a minimum total cost of 1481353 RMB for the remaining six days . The total cost per week is minimum 10300, 838RMB.

Table 3: Number of generators and their power for each type of time on the first day

Time								
			6-9	9-12	12-14	14-18	18-22	22-24
Мо	del							
	Number	0	1	1	1	1	1	0
1	Power		1800	800	1800	1700	1300	
	Number	5	5	5	5	5	5	5
2	Power	1480	1500	1320	1500	1500	1500	1360
	Number	0	8	7	8	7	7	2
3	Power		2000	2000	2000	2000	2000	2000
	Number	2	4	2	4	1	4	4
4	Power	1800	1925	1800	2675	1800	1800	1800

Table 4: Number and power of each type of generator for each period of the remaining six days

Time		0-6	6-9	9-12	12-14	14-18	18-22	22-24
Мо	del							
	Number	0	2	2	2	0	0	0
1	Power		1150	800	1800			_
	Number	5	5	5	5	5	5	5
2	Power	1000	1500	1480	1500	1480	1360	1360
	Number	2	8	8	8	8	8	2
3	Power	1200	2000	2000	2000	2000	2000	2000
	Number	4	4	0	4	1	4	4
4	Power	1800	1800	1800	2225	1800	1800	1800

From the data in the table, it is not difficult to see that the two-model generator and the four-model generator because of its low start-up capital, so the lower power demand 0-6 and 22-24 periods of time occupied the main number. Because of its low marginal cost, the three-model generator has been in the state of maximum output power during the intermediate time period with high

power demand. Such an arrangement can not only make the distribution of electricity more reasonable, but also minimize the start-up cost, so that the two time periods of convergence more smooth, in line with the actual situation in the topic.

### 6.3 Analysis of the results of minimum total cost problem

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In response to question one, the approach taken in this article is to deal with continuous relationship segmentation. Since the main variable connecting different time periods is the number of generators that have been turned on, it is divided into seven days a week, and dividing the day into seven times can simplify the continuous power generation process. The starting cost is high on the first day, the first time on the first day has a low demand but a high start-up cost when the generator is turned on, and then the starting cost at the beginning of

each day will be different because of the situation that is already open. In this paper, after simulating the switching condition of the generator for seven days, it can be found that according to the analysis method of the lowest per day, the optimization of the cost arrangement of the generator has been stabilized after the third day and the minimum cost result is optimized, so it can be fully believed that the model has reached the desired effect.

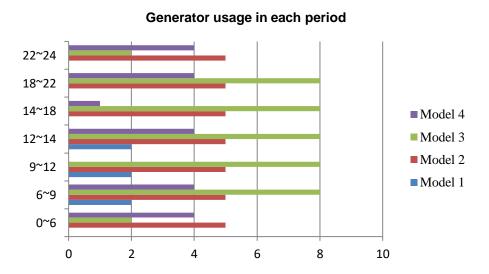


Fig.5:Number of units used for each engine on the first day

Fig.5 is derived from the optimal combination of the first day, and the usage of each model of engine shows that all four models of engines are in use except for the first and last stages.

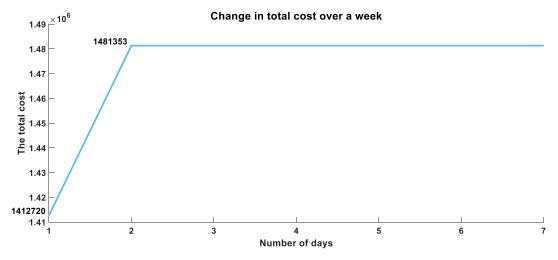


Fig.6:A totalcost change chart for a week

For the daily production costs of a week to chart the processing, to draw the following figure, from Figure 6 can be intuitively seen in a week in the first day of the production costs of the minimum of 1412720 RMB, the next day's cost increased to 1481353RMB, the next few days remain unchanged.

## 7. The establishment and solution of minimum total cost after reserved power

## 7.1 The establishment of the second model of minimum total cost after reserved power

### 7.1.1 Determining the target function

Minimum total cost after reserved power is still the problem about minimum cost and scheme, the target function to seek total cost and the relationship between the three costs and variables is unchanged. That is, the target function is still min W = G + B + Q

(1) The fixed cost of the four engines is

$$G = \sum_{i=1}^{4} \sum_{j=1}^{7} x_{ij} T_{j} G_{i}$$

(2) The cost of editor-in-chief when working with four types of engines is

$$B = \sum_{i=1}^{4} \sum_{j=1}^{7} (a_{ij} - M_i) x_{ij} T_j B_i$$

(3) The total start-up cost of starting the engine is

$$Q = \sum_{i=1}^{4} \sum_{i=1}^{7} \left[ D_{ij} + \left| D_{ij} \right| \right] / 2Q_{i}$$

(4) The difference in quantity between the  $\it i$ 

type of generator in the two-time period is

$$D_{ij} = \begin{cases} x_{ij} - x_{i(j-1)} & j = 2, 3, 4, 5, 6, 7 \\ x_{ij} - z & j = 1 \end{cases}$$

where z for the previous day  $x_{i7}$ 

### 7.1.2 Determining constraints

Since the title requires a 20% reserve of power generation capacity, and the minimum output power is only related to the generator itself, the reserved margin should be 80% of the maximum

output power, i.e. the maximum output power becomes 80% of the original

(1) The output power of the i type generator shall be less than the maximum allowable output power of the generator and greater than the minimum output power of the generator, i.e.

$$\begin{cases} 800 \le a_{1j} \le 1440 \\ 1000 \le a_{2j} \le 1200 \\ 1200 \le a_{3j} \le 1600 \\ 1800 \le a_{4j} \le 2800 \end{cases}$$

(2) In the j period of time, the i number of type of generators called should be less than the

total number of the type of generators that can be called is not less than zero, i.e.

$$\begin{cases} 0 \le x_{1j} \le 10 \\ 0 \le x_{2j} \le 5 \\ 0 \le x_{3j} \le 8 \\ 0 \le x_{4j} \le 4 \end{cases}$$

(3) The output power per hour of the generator set shall be greater than or equal to the required

power per hour during that period, i.e.

$$N_{j} \leq \sum_{i=1}^{4} \sum_{j=1}^{7} \left( a_{ij} x_{ij} \right)$$

### 7.1.3 Model of minimum total cost after reserved power

The model of minimum total cost after reserved power is basically the same as the problem one,

which is a multivariable nonlinear regression optimization model, and the main body of the model is not much different.

$$\min W = \sum_{i=1}^{4} \sum_{j=1}^{7} x_{ij} T_{j} G_{i} + \sum_{i=1}^{4} \sum_{j=1}^{7} + \left( a_{ij} - M_{i} \right) x_{ij} T_{j} B_{i}$$

$$\sum_{i=1}^{4} \sum_{j=1}^{7} \left[ D_{ij} + \left| D_{ij} \right| \right] / 2$$

### 7.2 Solution of the minimum total cost after reserved power model

The condition of minimum total cost after reserved power requires a 20% reserve of power generation, i.e. limiting the power generated by the generator set, so it is predicted that the number of units for all four types of generators will increase. For costs, the forecast is to reach an optimized state in the next few days, so the next few days should be equal. However, the results of the actual calculation show that the

number of stations of the generator is close to the expected value of this question, but the cost change is out of line with the expectation.

Based on the model lingo software to replace the data solverthe minimum cost per day for a week is 1481344, 1562351, 1600092, 1551320, 1560760, 1552081, 1563685RMB, Week Minimum total cost is 10871633 RMB, resulting in the number and power of each type of generator in each time period each day, the first and seventh days are given below.

Table 5: Number of generators and their power for each type of generator in each period of the first day

Time Model		0-6	6-9	9-12	12-14	14-18	18-22	22-24
	Number	4	10	10	10	9	9	9
1	Power	800	1440	880	1440	955.6	1440	933.3
	Number	5	5	5	5	5	5	5
2	Power	1200	1200	1200	1200	1200	1200	1200
	Number	0	3	3	5	2	2	0
3	Power		1600	1600	1600	1600	1600	1600
4	Number	1	4	3	4	4	4	2
	Power	1800	1950	1800	1900	1800	1960	1800

Table 6: Number of generators and their power for each type of generator during each period on the seventh day

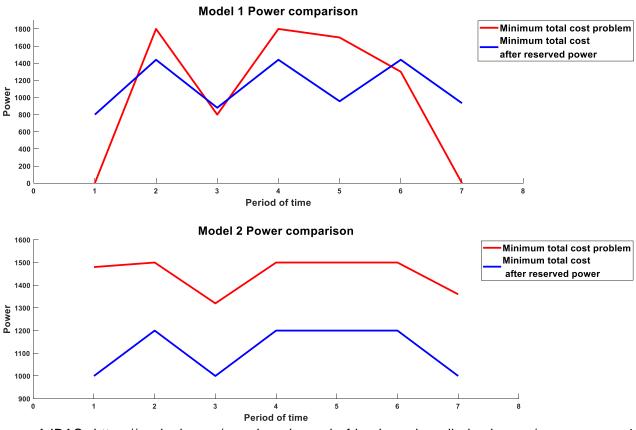
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Time								
		0-6	6-9	9-12	12-14	14-18	18-22	22-24
Мо	del							
	Number	10	10	10	10	10	10	3
1	Power	800	1440	800	1440	1020	1440	1440
	Number	5	5	5	5	5	5	5
2	Power	1000	1200	1000	1200	1200	1200	1200
	Number	2	3	3	3	1	1	0
3	Power	1200	1600	1600	1600	1600	1600	One
	Number	0	4	4	4	4	4	4
4	Power	One	1950	1800	2700	1800	2000	1920

### 7.3 Analysis of the results of minimum total cost after reserved power

In comparison with minimum total cost problem, the average output power of each generator set is less than the same combination of the problem in the case of unlimited, because the power generation margin is reserved, which is the same as the title. After several consecutive days of data calculation, we can find that the cost

has increased slightly to a stable, the number of different types of generator sets fluctuateless, basically to a stable state. And due to the reduction of maximum power, the use of the generator of model three decreased and the use of the generator of model one increased. This change is normal in order to meet the demand for electricity.



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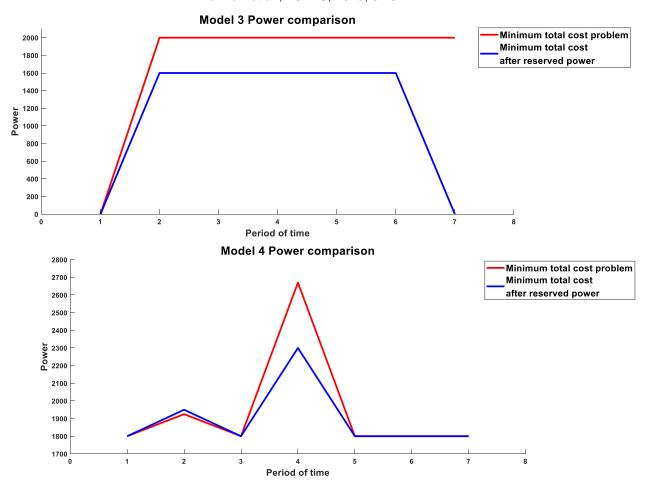


Fig. 7:Power changes in engines at different times

By comparing the figure above, we can see that the laws of output power of different models of generators are generally reduced, but slightly different at some stages.

Model 1:The first question is only in a few time periods where the power demand is too high, and the power is high, and in the rest of the case basically not called. Second question The amount of usage and power maintained at a high value throughout the whole stage, only because of the restrictions and slightly less than the power of the first question, proving that the marginal cost of the Model 1 generator is slightly higher, the same as the condition given by the title.

Model 2:The power in the first question is basically maintained at about 1.25 times the power in the second question, and the usage in

both questions basically reaches the maximum value, proving that the average cost of the Model 2 generator is minimal.

Model 3:The data in both the first and second questions have reached the maximum output power reflect the lower marginal cost of the Model 3 generator, but due to the higher fixed cost, the use of the Model 3 generator has a tendency to decrease due to the high fixed cost and the general increase in the use of the second question.

Model 4:The first question coincides with the middle part of the second question indicating that the marginal cost of the model 4 generator is too high to affect its use and does not rely on the output power of the Model 4 generator when the demand for power is low.

### 8. The evaluation of the model

### 8.1 Benefits of the model

- (1) The model takes into account the problem that the generator can continue to work during different time periods, and extends it to two natural days, so that the whole model can have continuity in the solution of the time period, and can be more accurate in line with objective facts.
- (2) The constraints and assumptions of the model are not conflicting and consider edgtosis.
- (3) The model provides a multi-constraint integer nonlinear combination optimization problem, generally concise and clear, convenient for programming.

### 8.2 Weaknesses of the model

- (1) There is a certain error in the assumption that the end and beginning of a single time period are too ideal.
- (2) The variables are less than the actual situation, the relationship between the variables is relatively single, it is not easy to simulate the accuracy of a longer period of time.

### 9. Model improvement and promotion

### 9.1 Improvements to the model

Model in the processing of generator start and shut down this process is too idealized, after consulting the data to know that the general generator start-up to check the water tank cooling water or antifreeze is satisfied, pull out the oil scale to see if the lubricant is missing, but also to carefully check the relevant components for the potential for failure. The output of the generator Air switch must be switched off before starting. Normal generator set diesel engine after start-up to go through 3-5 minutes of idle operation. And before each shutdown, you must gradually cut off the load, then turn off the generator set output air switch, and finally slow the diesel engine to idle state running about 3-5 minutes before stopping. These costs can be described as start-up costs in the question, but

the idle operation time accounts for about 10% of an hour, and the error is clearly not completely ignored.

So for both the start-up and shutdown processes this article can be set to 4 minutes, and the generator in this process cannot generate electricity. The expression of the actual power generation may be changed to

$$N = \sum_{i=1}^{4} \sum_{j=1}^{7} k_{ij} x_{ij} a_{ij}$$

Where, the value of  $k_{ij}$  is the parameter after taking into account the time factor, which can generally be expressed as

$$\frac{14}{15} |x_{ij} - x_{i(j-1)}|$$

This formula takes into account the situation where the generator can operate continuously, and also considers the time difference between the generator's inability to operate during start-up and shutdown, so the actual cost will be higher than the result of the model.

Considering that more detailed time segmentation can make data changes on the timeline more sensitive, the cost calculation will also have a greater change, only more simulation of the environment in the topic can be better through the model to get the most realistic optimal solution.

#### 9.2 Promotion of the model

The demand for electricity in each time period in the topic meets the general law of work, 0-6 points when sleeping less electricity consumption, 12-14 points for the peak period of activity power consumption. And the problem of generators can not only be studied or discussed in this emergency situation, in general, it can also be considered to use this model to promote calculations to save costs. The closer the problem to consider, the more common the

situation can be. The idea of segmentation and circulation is adopted when considering the number of days and the number of generator slots started continuously, so that the results of the calculation have a certain continuity and can solve the problem of the impact of multiple consideration symupes. The idea of popularization also has some enlightening effect on solving continuous optimization models. And this multi-constraint integer nonlinear combination optimization model and ideas can also be applied to other industries, that is, under the condition of limited resources, reasonable allocation of resources to get the corresponding requirements, while making the consumption minimal. This model has good application value and can be further promoted and studied.

#### 10. Conclusion

This paper comprehensively analyzes the possibility of opening schemes and the impact of three kinds of costs on the total cost, and establishes a multi-constraint integer nonlinear optimization model. In order to minimize the total cost of a week, that is, the total cost of a day, a minimum model of fixed cost, marginal cost and start-up cost is established. By means of dynamic programming, the scheme with the minimum cost of seven days, that is, the scheme with the minimum total cost of one week, was calculated. Can rationalize the arrangement of the use of the machine, so that the machine to achieve the maximum utilization while reducing the cost, to obtain the maximum profit, the use of the factory machine arrangement has a very high guiding significance. After changing the output power of the generator, the constraint conditions of the output power are adjusted based on the model of the above problems, and a new model of electricity demand and generation is obtained.In this paper, the model is extended to other

problems of continuous segmentation, and the conclusion in line with living habits is drawn by analyzing the data. After consulting the data, the improvement scheme of the model on real life is put forward, which can be universally used to solve such problems.

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