



CT Image Reconstruction Method Based on Filtered Back Projection Algorithm

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ABSTRACT

This paper used the relevant knowledge of electronic computer tomography image reconstruction to solve the problem of CT system parameter calibration and imaging algorithm. Through geometric analysis, a target optimization model was established. The parallel beam filtering back-projection algorithm was used to solve the problem based on the information received by the detector unit. The image reconstruction model was used to obtain the position information of the medium. Finally, image stability was verified by noise processing.

Keywords: Filtered Back Projection; Wavelet principle CT; System parameters

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1. Introduction

Computerized tomographic scanning (CT) is a kind of method which can be used to image the samples of biological tissues and engineering materials and analyze the structural characteristics of the samples without destroying the samples. It is necessary to calibrate the parameters of the CT system with the help of known samples, which are called templates, and then image the position samples according to the obtained parameters [1].

For imaging methods, Donono et al. [2] and Candes et al. [3] proposed a new information acquisition theory in 2006, pointing out that if the signal is sparse in a specific change domain, it needs little information sampling, and the original signal can be recovered by solving the optimization problem. Gou et al. [4] Studied split Bregman's CT image reconstruction algorithm, which can distinguish material, but failed to study the influence of image artefacts. Based on the theory of X-ray attenuation and random change, this paper uses a filter back-projection algorithm to construct the shape of the medium. The problem solved in this paper comes from the problem an of the undergraduate group of 2017 National College Students' mathematical modeling competition of social cup of higher education. Please refer to the topic information for relevant data.

2 The calibration method of CT system parameters

2.1 Detector cell spacing model

According to the Beer-Lambert law^[8], there is $I = I_0 \cdot e^{-\mu x}$, and the logarithmic transformation is carried out to obtain that the ratio of the absorption rate ξ to the ray penetration thickness x

$$s.t. \begin{cases} d = \frac{d_1 + d_2}{2} \\ l_i = 2\sqrt{R^2 - d_i^2} \end{cases} \quad (i = 1, 2, \dots, n)$$

2.2 Establishing X-ray direction model based on analytic geometry principle

Let R_{ij} be the value of the detector j at the angle i of X-ray. Assuming that the detection value at

is equal to the attenuation coefficient of the sample, that is, $\xi = \mu x$. Because μ is constant when the distance between P_0 and the centre of the circle approaches 0, the penetration thickness l_0 is approximately equal to the diameter D of the calibration template, so the ratio equation of the penetration thickness and the calibration tem-

$$\frac{\xi_0}{D} = \frac{\xi_1}{l_1} = \frac{\xi_2}{l_2}$$

plate is obtained

Where ξ_i is the absorptivity of ray i , l_i is the penetration thickness of ray i .

The objective function is to minimize the difference between the absorption rates of adjacent rays with the maximum absorption rate.

$$\min \sum_{i=1}^n (\xi_{i1} - \xi_{i2})^2$$

In the circular model, the difference of the circular radius and the penetration length can form a square. The Pythagorean theorem can be obtained as follows:

$$l_i = 2\sqrt{R^2 - d_i^2}$$

tained as follows:

Combining projection formula and ξ_i size, d_1 and d_2 with minimum difference can be obtained. Because the ray passing through the centre of the circle does not necessarily exist, the influence of this error can be eliminated by averaging d_1 and d_2 . Thus, the unit distance of

$$d = \frac{d_1 + d_2}{2}$$

the detector is obtained as follows:

In conclusion, the nonlinear programming model

$$\min \sum_{i=1}^n (\xi_{i1} - \xi_{i2})^2$$

is obtained:

this time is R_{ij} , the distance between the sensor j and the detector m is $(j-m)d$, because the

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

elliptic equation is we can get the

multiple corresponding sets of penetration distance formulas as follows:

$$l_{js} = 2b \sqrt{1 - \left(\frac{(j-m)d}{a} \right)^2} \quad (j, s = 1, 2, \dots, n)$$

Let the fitting function $R = al + b$, in which a and b make the sum of the squares of the residuals the minimum that is:

$$\min \sum_{i=1}^n [R_{ij} - (al_{js} + b)]^2$$

The least-square method is used to solve the partial derivative of a and b is equal to 0, and the gain coefficient and optimize the problem. The following results are obtained:

$$\begin{bmatrix} n & \sum_{i=1}^n l_{si} \\ \sum_{i=1}^n l_{si} & \sum_{i=1}^n l_{si}^2 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n R_{si} \\ \sum_{i=1}^n R_{si} l_{si} \end{bmatrix} \quad (i = 1, 2, \dots, n)$$

The linear relationship between the maximum group of detection values is obtained value R_{im} and the penetration length l_{si} in each

$$R_{im} = al_{si} + b$$

Let the X-ray direction θ_i be defined as the angle between the X-ray direction and the x-axis direction. According to the geometry knowledge, the relation between the course θ_i of the available X-ray and the projection length is deduced. Therefore, we can set:

$$\begin{cases} y_i = \frac{l_i}{2} \sin \theta_i \\ x_i = \frac{l_i}{2} \cos \theta_i \end{cases} \quad (i = 1, 2, \dots, n)$$

By substituting the above substitutions into the analytical expression of the ellipse, the analytical expression of X-ray direction can be obtained as follows:

$$\theta_i = \arctan \left(\frac{\sqrt{\frac{4}{l_{si}^2} - \frac{1}{a^2}}}{\sqrt{\frac{1}{b^2} - \frac{1}{a^2}}} \right) \quad (i = 1, 2, \dots, n)$$

3. CT image reconstruction method

Central slice theorem. The absorption intensity function $f(x, y)$, the projection function $P_0(\rho)$ in a specific direction, is the two-dimensional Fourier function $F(\rho, \theta)$ in the original

density function $f(x, y)$, the value of (ρ, θ) in the straight line in the same direction of the plane [5].

Using [6] the two-dimensional Fourier transform in polar coordinate form as the following formula:

$$\mu(x) = \int_0^{2\pi} \int_0^{+\infty} \bar{\mu}(\omega\phi) e^{i2\pi x \cdot (\omega\phi)} \omega d\omega d\phi$$

Where $\bar{\mu}(\omega\phi) = \iint_{\mathbb{R}^2} \mu(x) e^{i2\pi x \cdot (\omega\phi)} dx$

According to the above Fourier transform and central slice theorem, the reconstruction formula is obtained as follows:

$$\mu(x) = \int_0^\pi \left[\int_{-\infty}^{+\infty} \tilde{p}(k, b) |w| W(\omega) e^{-i2x\gamma w} d\omega \right]_{\gamma=x\phi} d\varphi$$

Where $\phi = (x, y)$ represent any point, $\tilde{p}(\omega, \varphi) = \int_{-\infty}^{+\infty} p(k, b) e^{-i2x\gamma w}$, $W(\omega)$ represents

a window function. According to the window function and the constraint of any point, the reconstruction function of filtered back projection is obtained as follows:

$$\mu(x) = \int_0^\pi \left[\int_{-\infty}^{+\infty} \tilde{p}(\omega, \varphi) |w| W(\omega) e^{-i2x\gamma w} d\omega \right]_{\gamma=x\phi} d\varphi$$

Where $\tilde{p}(\omega, \varphi)$ is $\int_{-\infty}^{+\infty} \tilde{p}(\gamma, \varphi) e^{-i2x\gamma w} d\gamma$.

Noise refers to the random fluctuation of CT value above and below the average value in the image of a uniform object. Its value can be expressed by SD of the standard deviation of CT

value. The SD value of the FBP inversion formula is small; the change of CT value curve is small; the noise of the reconstructed image is small. Otherwise, the noise is significant.

$$SD = \sqrt{\frac{\sum_{i=1}^N (X_i - \bar{X})^2}{N-1}} \quad (i = 1, 2, \dots, N)$$

Where N represents the total number of pixels, X_i is the CT value of each pixel, \bar{X} is the average of all pixels. The smaller the thickness of the slice, the higher

the noise; CT noise and dose-dependent, the larger the dose, the smaller the noise. Let B be the attenuation factor and C constant coefficient. According to the Braux formula, the correlation can be obtained as follows:

$$\sigma = C \sqrt{\frac{B}{W^2 h D_0}}$$

Where W represent pixel width, h represent the fault thickness, D_0 describe the initial ray intensity. If the threshold value is too small, the denoised image still has noise. On the contrary, if the threshold value is too large, the essential image features will be filtered out, causing deviation.

Therefore, the soft threshold processing is to compare the absolute value of the signal with the threshold value. When the total value of the data is less than or equal to the threshold value, the noise will be zero; when the data point is higher than the threshold value, it will shrink to zero and become the difference between the point value and the threshold value^[7].

$$\text{Noise treatment} \begin{cases} 0 & (|\eta| \geq \sigma_{\tilde{v}}) \\ \sigma_{\tilde{v}} - \eta & (|\eta| \leq \sigma_{\tilde{v}}) \end{cases}$$

4. Denoising of unknown media receiving information

Take 0.05 as the step, take different noise thresholds between 0-0.3, and examine the image after noise removal. The Fig. below shows

the three-dimensional logarithmic coordinate schematic image of the received information in each detection direction after noise reduction, as shown in Fig. 1.

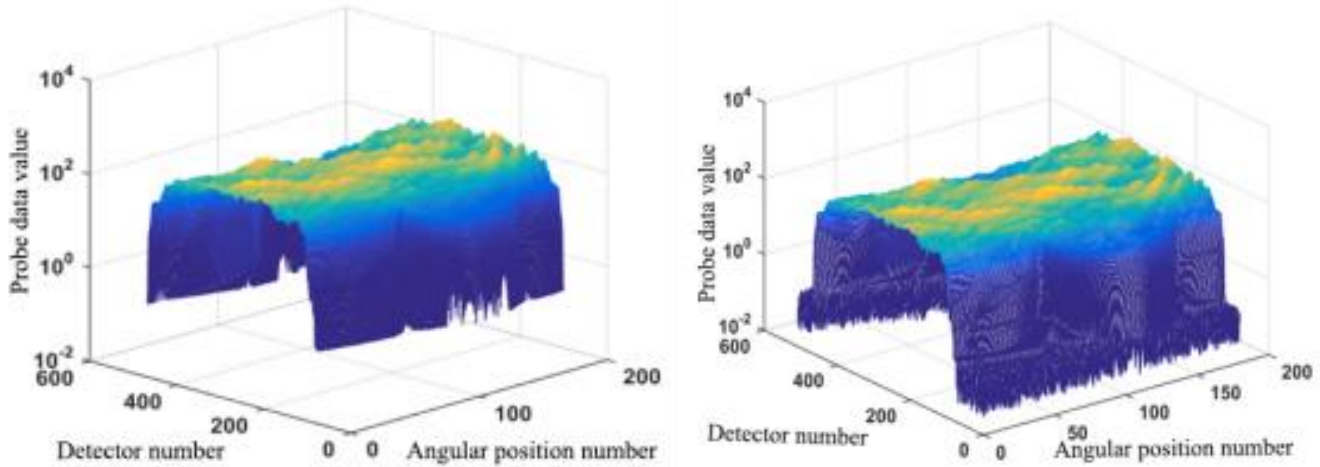


Fig. 1 Three-dimensional logarithmic coordinates of the received information

5. The solution of calibration parameters

We solve the above-mentioned nonlinear programming model and take the maximum absorption rate and the minimum difference between the absorption rates of the left and right adjacent rays as the objective to obtain the absorption rates of the intermediate rays and the adjacent two rays so that the intermediate rays are the rays passing through the long axis of the ellipse

so that the obtained results are the optimal solution. That is to say, the unit spacing of the detector is 0.2768mm.

The coordinates of the rotation centre of the CT system are (- 9.2996, 5.5520) and the 180 directions of X-ray used by the CT system are [- 60.2152 °, 118.7725 °]. Some results are as Table 1. The specific transformation is shown in Fig.2.

Table1 180 X-ray directions

Frequency	1	2	3	180
Angle	-60.2152°	-58.8587°	-58.3008°	118.7725°

6. Simulation reconstruction experiment

Combined with the received information of the unknown media in Annex 5, the filter back-projection algorithm (FPB) is used to obtain the absorption rate, geometry, and position of the unknown media in the square template, and the resulting image is denoised to obtain the following reconstructed image.

Unknown geometry and location of media. Acc-

ording to the absorption rate results, the geometric shape before and after noise removal is represented as 256 × 256 pixel image (the square template is divided into 256 columns and 256 rows). There are many interferences, such as gross noise and blank points in the left image. After noise removal, a relatively flat image is obtained, as shown in the right figure. The geometry and location are shown in the Fig.2.

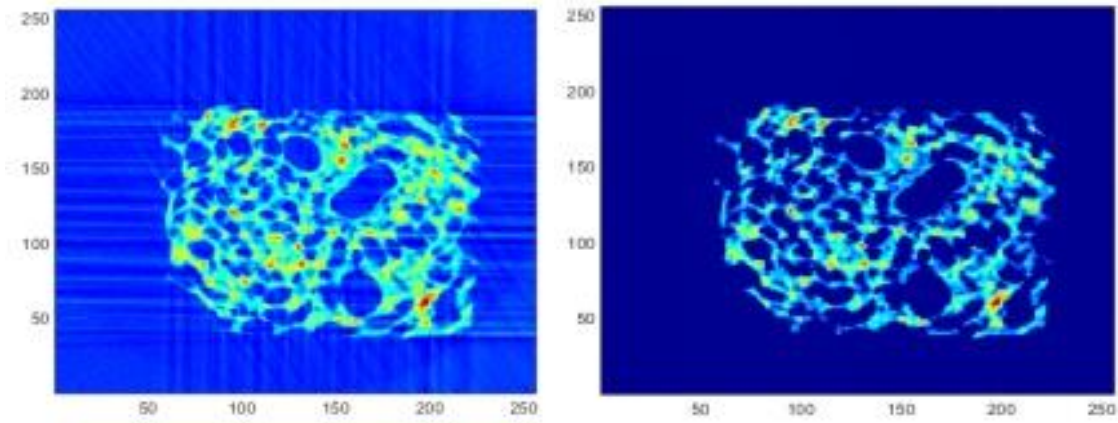


Fig.2 Comparison of geometry before and after noise removal

The absorptivity of the unknown medium. We use an image reconstruction algorithm to get a 256×256 absorbance matrix. In order to make the relationship between absorptivity and graph

more intuitive, the following three-dimensional graph is drawn. The height of the image represents the absorption rate.

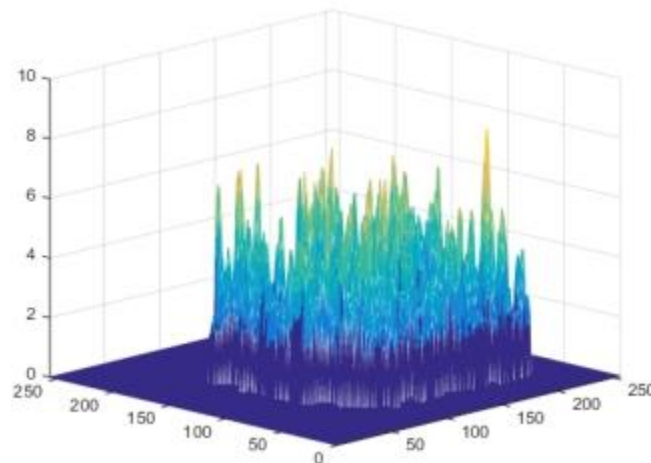


Fig.3 Absorbance size across geometry

The absorption rate of ten specific positions. The matrix is mapped to the square template, and

the specific absorption rate is obtained, as shown in the Table 2.

Table 2 absorbance size at ten locations

Coordinates	Absorptivity	Coordinates	Absorptivity
(10.0,18.0)	0	(50.0,75.5)	3.2351
(34.5,25.0)	2.8091	(56.0,76.5)	6.1054
(43.5,33.0)	7.0353	(65.5,37.0)	0
(45.0,75.5)	0	(79.5,18.0)	8.1176
(48.5,55.5)	0.0101	(98.5,43.5)	0

The coordinate position in the table is the coordinate point given in Annex IV, which is a plane rectangular coordinate system established by two sides of a square.

7. Stability analysis

Analyze the stability of the calibration results when there is noise on the receiver of the calibration model, artificially add the Gaussian white noise with the mean value of 0 and the standard deviation of 3 on the received data (Annex II

data), which obeys the normal distribution and then calibrate the template according to this. The

result of Gaussian noise processing is shown in Fig.4.

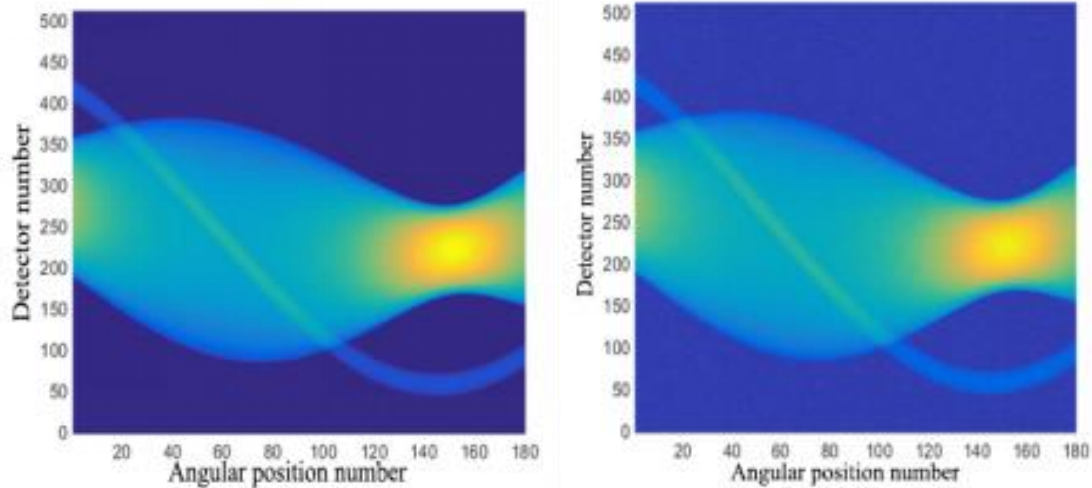


Fig.4 Comparison before and after receiving information noise

Note: the left image is the received information image before adding noise, and the right image is processed by Gaussian noise.

According to the above Figure, it can be found that the edge of the image has a micro saw tooth shape compared with the left image without noise. The unabsorbed part has a lot of burrs and is not smooth, so it can be concluded that the influence of Gaussian noise is excellent. It has a particular reference value for the later angle analysis.

8. Conclusion

This article focuses on CT system parameter calibration, imaging, and model construction issues. Through geometric analysis, a target planning model is established, and a parallel beam filtering back-projection algorithm is adopted to analyze the geometric algorithm. The parameters such as the rotation direction of the CT system and the determination of the geometric information of the unknown medium were obtained. Finally, a new template was designed to improve the accuracy and stability of the system parameters. For the problem of CT image reconstruction, we established a parallel beam filtered back-projection reconstruction model. Using the principle of Fourier transform, the RL filter is used in the algorithm design. The projection data is first Fourier transformed, and the inverse Fourier processed after filtering. Reconstruction of the image. The position, absorptivity, geometric

information and other information of the de-noised image were obtained, and the absorptance data of the specified ten positions were given.

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