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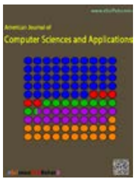

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Review of Mathematical Approach to Engineering Problems

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ABSTRACT

Mathematics is widely used in every engineering fields. In this paper, several examples of applications of mathematics in mechanical, chemical, optimization and electrical engineering are discussed. Laplace transform mathematical tool is applied to solve problems. Applications here are the real ones found in the engineering fields, which may not be the same as discussed in many mathematics text books. The purpose of this paper is to relate mathematics to engineering field.

Keywords: Laplace transform ; Mechanical; Mechanical; Electrical; Optimization

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1.Introduction

Mathematical modeling has always been an important activity in science and engineering. The formulation of qualitative questions about an observed phenomenon as mathematical problems was the motivation for and an integral part of the development of mathematics from the very beginning. Although problem solving has been practiced for a very long time, the use of mathematics as a very effective tool in problem solving has gained prominence in the last 50 years, mainly due to rapid developments in computing. Computational power is particularly important in modeling engineering systems, as the physical laws governing these processes are complex. Besides heat, mass, and momentum transfer, these processes may also include chemical reactions, reaction heat, adsorption, desorption, phase transition, multiphase flow, etc. This makes modeling challenging but also necessary to understand complex interactions.

All models are abstractions of real systems and processes. Nevertheless, they serve as tools for engineers and scientists to develop an understanding of important systems and processes using mathematical equations. In all engineering context, mathematical modeling is a prerequisite for: design and scale-up; process control; optimization; mechanistic understanding; evaluation/planning of experiments; trouble shooting and diagnostics; determining quantities that cannot be measured directly; simulation instead of costly experiments in the development lab; feasibility studies to determine potential before building prototype equipment or devices.

Mathematics is the background of every engineering fields. Together with physics, mathematics has helped engineering develop. Without it, engineering cannot evolved so fast we can see today. Without mathematics, engineering cannot become so fascinating as it is now. Linear algebra, calculus, statistics, differential equations and numerical analysis are taught as they are important to understand many engineering subjects such as fluid mechanics, heat transfer, electric circuits and mechanics of materials to name a few. However, there are many complaints from the students who find it difficult to relate mathematics to engineering. After studying differential equations, they are expected to be able to apply them to solve problems in heat transfer, for example. However, the truth is different. For many students, applying mathematics to engineering problems seems to be very difficult. Many examples of engineering applications provided in mathematics textbooks are often too simple and have assumptions that are not realistic. See([8],[9],[10],[11]) for a good textbook which discusses mathematical modelling with real life applications. A lot of problems solved using Maple and MATLAB are given in [12,13,14]. The purpose of this paper is to show some applications of mathematics to various engineering fields. The applications discussed do not need advanced mathematics so they can be understood easily.

2. Mathematical Approach to Engineering Problems

In this section we discussed four engineering problems, first problem is about electrical circuit problem, the second on the mixing solutions in two tanks , Optimization Problem and the last on the stability problem .

2. 1. Electrical circuit

To find the current in the RC-circuit in Figure1. If a single rectangular wave with voltage is applied as a input . The circuit is assumed to be quiescent before the wave is applied. The input in terms of unit step function is given by

$$v(t)=V_0\left[u(t-a)-u(t-b)\right]$$

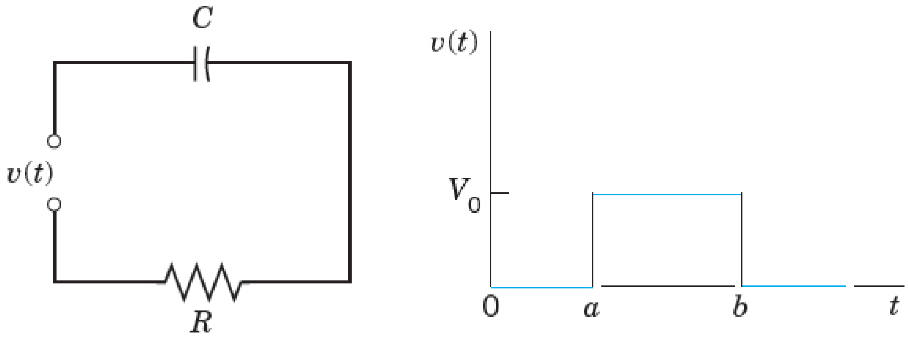

Figure.1 RC-circuit with input $v(t)$

Applying KVL to above circuit, we get

$$Ri(t)+\frac{1}{C}\int_0^t i(\tau)d\tau = V_0\left[u(t-a)-u(t-b)\right] \quad (1.1)$$

Taking Laplace transform, we get

$$RI(s)+\frac{1}{C}\frac{I(s)}{s} = V_0\left[\frac{e^{-as}}{s}-\frac{e^{-bs}}{s}\right] \quad (1.2)$$

$$\left(R+\frac{1}{sC}\right)I(s) = V_0\left[\frac{e^{-as}}{s}-\frac{e^{-bs}}{s}\right] \quad (1.3)$$

$$I(s)=\frac{V_0}{R}\left[\frac{e^{-as}}{\left(s+\frac{1}{RC}\right)}-\frac{e^{-bs}}{\left(s+\frac{1}{RC}\right)}\right] \quad (1.4)$$

Taking inverse Laplace transform, we get

$$i(t)=\frac{V_0}{R}\left[e^{-\frac{1}{RC}(t-a)}u(t-a)-e^{-\frac{1}{RC}(t-b)u}u(t-b)\right] \quad (1.5)$$

is the required current in the *RC*-circuit.

2.2. Mixing Problem Involving Two Tanks

Tank in **Figure.2** initially contains 100 gal of pure water. Tank initially contains 100 gal of water in which 150 lb of salt are dissolved. The inflow into is from and containing 6 lb of salt from the outside. The inflow into is 8 gal/min from. The outflow from is, as shown in the figure. The mixtures are kept uniform by stirring. Our aim is find the salt contents $x_1(t)$ and $x_2(t)$ in tanks T_1 and T_2.

2.3. Optimization Problem (Minimization of drag-to-lift ratio)

Airplane pilots share a challenge with flying birds: How far can they go. What is their range for a fixed amount of fuel? Still better, can they maximize their range? It turns out that for a given amount of fuel, the speed that maximizes the range is the one that maximizes the aerodynamic quantity, called the lift-to-drag ratio, or, conversely, minimizes its inverse, the drag-to-lift ratio.

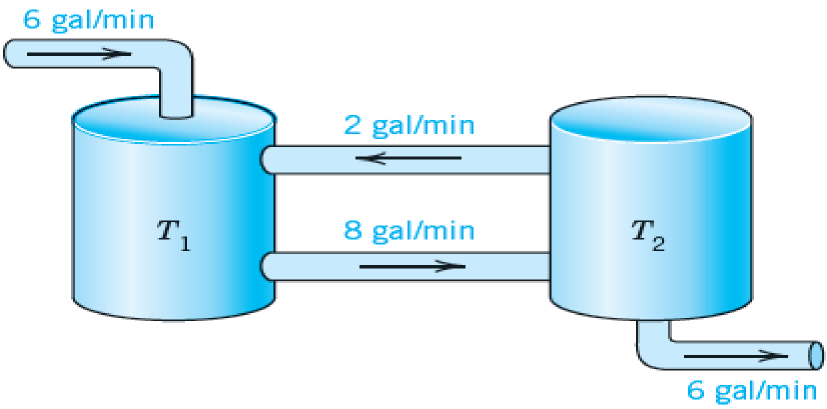

Figure.2 Mixing Problem Involving Two Tanks

Setting up the model.

Time rate of change = *Inflow*/min - *Outflow*/min

For tank T_1 **:** $x_1'(t)=\frac{2}{100}x_2(t)-\frac{8}{100}x_1(t)+6$ (2.1)

For tank T_2 **:** $x_2'(t)=\frac{8}{100}x_2(t)-\frac{2}{100}x_1(t)$ (2.2)

with initial conditions are $x_1(0)=0, x_2(0)=150$.

By taking the Laplace transform we get

$$(s+0.08)L[x_1(t)]-0.02L[x_2(t)]=\frac{6}{s} \quad (2.3)$$

$$(-0.08)L[x_1(t)]+0.08L[x_2(t)]=150 \quad (2.4)$$

We solve this algebraically for $L[x_1(t)]$ *and* $L[x_2(t)]$ and we write the solutions in terms of partial fractions,

$$L[x_1(t)]=\frac{9s+0.48}{s(s+0.12)(s+0.04)}=\frac{100}{s}-\frac{62.5}{(s+0.12)}-\frac{37.5}{(s+0.04)} \quad (2.5)$$

$$L[x_2(t)]=\frac{150s^2+12s+0.48}{s(s+0.12)(s+0.04)}=\frac{100}{s}+\frac{125}{(s+0.12)}+\frac{75}{(s+0.04)} \quad (2.6)$$

By taking the inverse transform we arrive at the solution

$$x_1(t)=100-62.5e^{-0.12t}-37.5e^{-0.04t} \quad (2.7)$$

$$x_2(t)=100+125e^{-0.12t}+75e^{-0.04t} \quad (2.8)$$

are the required salt contents in T_1 and T_2

Figure. 3 a typical jet with a free-body diagram superposed. The plane is climbing at an angle, α, at a speed, V, relative to the ground. The climb or flight direction angle, α, is zero for level flight, and positive for ascending flight and negative for descending flight. The free-body diagram shows the forces that act to support the plane and move it forward, as described in the aerodynamic literature. The plane's weight, W, is supported by a lift (force), L, that is perpendicular to the flight path. The engines provide a thrust, T, that moves the plane along the flight path by overcoming the drag (force), D, that also acts along the flight path, albeit it in a direction that retards flight. The plane's wing has a surface area, S, and span, b.

Figure. 3. A typical jet with a superposed free-body diagram showing the aerodynamic forces acting.

Lift force and drag force is given by

$$L = \frac{1}{2}\rho S V^2 C_L \qquad (3.1)$$

And

$$D = \frac{1}{2}\rho S V^2 C_D \qquad (3.2)$$

where C_L and C_D are the corresponding *lift* and *drag coefficients*. (We should note that the drag-velocity relation is more complicated when planes fly closer to the speed of sound, due to drag produced by compressibility effects either on rapidly rotating propellers or on the wings of jet aircraft). The makeup of the C_L and C_D coefficients and their relationship provide, the complexity we will see in our search for an optimum flight speed. But first we need to do a little equilibrium analysis because taken superficially, equations. (3.1–3.2) suggest that the drag-to-lift ratio *L/D* is independent of the speed *V*, so how could it be minimized with respect to *V*?

We sum the forces superposed on the plane in Figure . 3 in the *x* and *y* directions:

$$\sum F_x = T\cos\alpha + L\sin\alpha + D\cos\alpha = 0 \qquad (3.3)$$

And

$$\sum F_t = T\sin\alpha + L\cos\alpha - D\sin\alpha - W = 0 \qquad (3.4)$$

If the climb angle, *α*, is assumed to be small, Using the approximations

$$\left(\sin\alpha = \alpha - \frac{\alpha^3}{6} + \ldots\ldots\ldots \quad and \quad \cos\alpha = 1 - \frac{\alpha^2}{2} + \ldots\ldots\ldots\right)$$

equations (3.3-3.4) can be simplified and solved to show that the lift *L* is,

$$L \cong \frac{W}{1+\alpha^2} \cong W \qquad (3.5)$$

which means that the *drag-to-lift ratio* is simply

$$\frac{D}{L} \cong \frac{D}{W} \qquad (3.6)$$

Equation(3.5) clearly shows that the lift force supports the plane's weight, while equation (3.6) provides a speed-dependent ratio of the drag force to the weight. Now we return to the drag coefficients because that is the logical step for casting the *D/L* ratio in terms of the plane's speed, *V*.

It turns out that the drag coefficient is expressed as a sum of two terms,

$$C_D = C_{D_0} + \frac{kSC_L^2}{\pi b^2}. \qquad (3.7)$$

The first term represents the *parasite* or *friction drag* caused by shear stresses resulting from the air speeding over and separating from the wing. The second term is the *induced drag* : it is independent of the air viscosity and is created by wings of finite span (i.e., real wings!) because of momentum changes needed to produce lift, according to Newton's second law. Note that the induced drag is proportional to the square of the lift coefficient, C_L^2.

Now we can combine equations. (3.1) and (3.6) to write the drag-to-lift ratio as

$$\frac{D}{L}=\frac{\rho SV^2C_D}{2W} \qquad (3.8)$$

after which we can further combine. equations. (3.1), (3.5) and (3.7) to rewrite (3.5) . (3.8) as

$$\frac{D}{L}=C_{01}V^2+C_{02}V^{-2} \qquad (3.9)$$

With the constants C_{01} *and* C_{02} defined as

$$C_{01}=\frac{\rho SC_{D_0}}{2W},\ C_{02}=\frac{2kW}{\pi\rho b^2} \qquad (3.10)$$

Thus, the objective function or *cost* for this optimization problem is defined in equations. (3.9), and its coefficients as presented in equation (3.10) are simply constants reflecting the values of the problem's physical parameters: *ρ*, *S*, *W*, the wing span, *b*, the parasite drag coefficient, C_{D0} and a dimensionless shape constant, *k* .

The extreme value of this unconstrained optimization problem is then found by the standard calculus approach, that is,

$$\frac{d}{dV}\left(\frac{D}{L}\right)=2C_{01}V-2C_{02}V^{-3}=0 \qquad (3.11)$$

which has the following extreme value:

$$\left(\frac{D}{L}\right)_{\min}=2\sqrt{C_{01}C_{02}}\ \ at\ \ V_{\min}=2\left(\frac{C_{02}}{C_{01}}\right)^{1/4}. \qquad (3.12)$$

With the aid of equation (3.10), the minimum drag-to-lift ratio can then be written in its final form

$$\left(\frac{D}{L}\right)_{\min}=2\sqrt{\frac{kSC_{D_0}}{\pi b^2}} \qquad (3.13)$$

This is a classical result in aerodynamics. Further, it is also easily demonstrated at this minimum *D/L* ratio occurs only when the parasite drag and the induced drag are equal and, consequently, independent of the plane weight *W*.

2.4. Stability Problem

Many smaller portable tape recorders have a capacitor microphone built in, since such a system is simple and robust. It works on the principle that if the distance between the plates of a capacitor changes then the capacitance changes in a known manner, and these changes induce a current in an electric circuit. This current can then be amplified or stored. The basic system is illustrated in Figure.4 There is a small air gap (about 0.02 mm) between the moving diaphragm and the fixed plate. Sound waves falling on the diaphragm cause vibrations and small variations in the capacitance *C* ; these are certainly sufficiently small that the equations can be linearized.

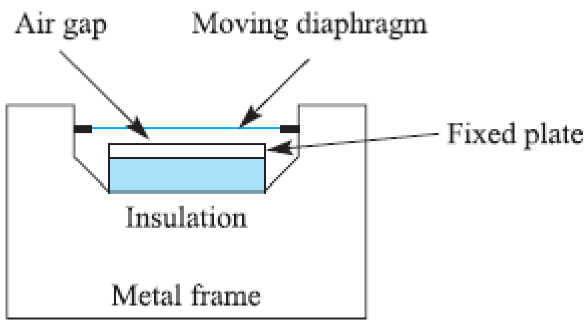

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Figure.4 Capacitor microphone

We assume that the diaphragm has mass *m* and moves as a single unit so that its motion is one-dimensional. The housing of the diaphragm is modelled as a spring and-dashpot system. The plates are connected through a simple circuit containing a resistance and an imposed steady voltage from a battery. **Figure** illustrates the model. The distance *x* (*t*) is measured from the position of zero spring tension, *F* is the imposed force and *f* is the force required to hold the moving plate in position against the electrical attraction.

The mechanical motion is governed by Newton's equation

$$m\frac{d^2x}{dt^2}+\lambda\frac{dx}{dt}+x+f-F=0 \qquad (4.1)$$

and the electrical circuit equation gives

$$E=RI+\frac{q}{C} \qquad (4.2)$$

The variation of capacitance *C* with *x* is given by the standard formula

$$C=\frac{C_0a}{x+a} \qquad (4.3)$$

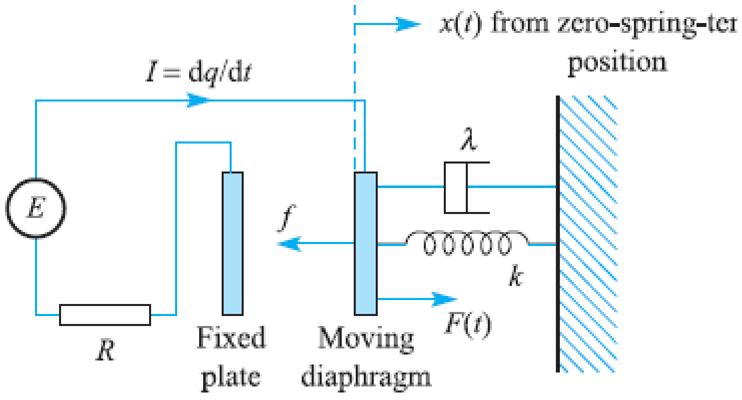

Figure.5 Capacitor microphone model.

where *a* is the equilibrium distance between the plates. The force *f* is not so obvious,

$$f = \frac{1}{2}\frac{q^2}{C_0 a} \quad (4.4)$$

It is convenient to write the equations in the first-order form

$$v = \frac{dx}{dt} \quad (4.5)$$

$$m\frac{dv}{dt} + \lambda v + x + f - F = 0 \quad (4.6)$$

$$R\frac{dq}{dt} = -\frac{q(a+x)}{C_0 a} + E \quad (4.7)$$

Furthermore, it is convenient to non-dimensionalize the equations. With distance and velocity, for the time and the charge using standard non-dimensionalization procedure by neglecting prime,

$$\tau = \frac{t}{\tau_1},\ X = \frac{x}{a}, V = \frac{v}{ka/\lambda},\ Q = \frac{q}{\sqrt{2C_0 ka^2}}$$

And equations are

$$X' = \frac{RC_0 k}{\lambda} V \quad (4.8)$$

$$\frac{R}{C_0 m\lambda} V' + X + V + Q^2 = \frac{F}{ka} \quad (4.9)$$

$$Q' = -Q(1+X) + \frac{EC_0}{\sqrt{2C_0 ka^2}} \quad (4.10)$$

There are four non-dimensional parameters: the external force divided by the spring force gives the first, *G* = *F*/*ka* ; the electrical force divided by the spring force gives the second, $D^2 = \frac{E^2 C_0^{\,2}}{2C_0 ka^2}$; and the remaining two are $A = \frac{RC_0}{\lambda}$ and $B = \frac{R}{C_0 m\lambda}$

The final equations are

$$X' = AV,\ \ BV' + X + V + Q^2 = G \ \ and \ \ Q' = -Q(1+X) + D \quad (4.11)$$

In equilibrium, with no driving force, *G* = 0 and $V = X' = V' = Q' = 0$, so that

$$\left.\begin{array}{l}(Q^2 + X) = 0 \\ Q(1+X) = -D\end{array}\right\} \quad (4.12)$$

on eliminating *Q*, weget

$$X(1+X)^2 = -D^2 \quad (4.13)$$

There are two physically satisfactory equilibrium solutions $-\frac{1}{3} > X < 0$ and $-1 > X < -\frac{1}{3}$, and the only question left is whether they are stable or unstable. Using standard stability analysis

Get the only solution that can possibly be stable is the one for which $X > -\frac{1}{3}$ and other solution is unstable.

Having established the stability of one of the positions of the capacitor diaphragm, the next step is to look at the response of the microphone to various inputs. The characteristics can most easily be checked by looking at the frequency response, which is the system response to an individual input $G = b\, e^{jwt}$ as the frequency ω varies. This will give information of how the electrical output behaves and for which range of frequencies the response is reasonably flat. The essential point of this example is to show that a practical vibrational problem gives a stability problem.

3. Conclusions

In this paper, four of applications of mathematics for different engineering fields have been presented. The problems are from real life and solved different techniques. It is expected that the problems presented in this paper can motivate reader to understand mathematics better. Mathematics should be enjoyable as it has helped engineering evolved.

References

[1] Gere, J.M. and Timoshenko, S.P., Mechanics of Materials, Third SI Edition. Dordercht: Springer Science Business Media, 1991.

[2] Popov, E., Engineering Mechanics of Solids. New Jersey: Prentice-Hall, 1990.

[3] J. E. Connor, J.E. and and Faraji, S., Fundamentals of Structural Engineering. Berlin Heidelberg: Springer-Verlag, 2012.

[4] Hjelmstad, K.D., Fundamentals of Structural Mechanics, Second Edition. New York: Springer-Verlag, 2005.

[5] White, R.E. and and Subramaniam, V.R., Computational Methods in Chemical Engineering with Maple. Springer-Verlag. Berlin Heidelberg, 2010.

[6] Keil, F., Mackens, W., Vo, H. And Werther, J., Scientific Computing in Chemical Engineering. Springer-Verlag, Berlin Heidelberg, 1996.

[7] Caldwell, J. and Ram, Y.M., Mathematical Modelling, Springer Science Business Media. Dordercht, 1999.

[8] Braun, M., Differential Equations and Their Applications, Springer Science Business Media. New York, 1993.

[9] Erwin Kreyszing., Advanced Engineering Mathematics : John Wiley & Sons, 2014.

[10] Glyn James., Advanced Modern Engineering Mathematics: Pearson Education Limited, 2011.

[11] Popov, E., Engineering Mechanics of Solids. New Jersey: Prentice-Hall, 1990.

[12] Jacobsen, R.T., Penoncelo, S.G. and Lemmon, E.W., Thermodynamic Properties of Cryogenic Fluids. Springer Science Business Media, New York, 1997.

[13] Reid, R.C., Prausnitz, J.M. and Poling, B.E., The Properties of Gases and Fluids. McGraw-Hill Inc., New York, 1987.

[14] Gander, W. And Hrebicek, J., Solving Problems in Scientific Computing Using Maple and MATLAB. Springer-Verlag, Berlin Heidelberg, 2014

[15] K. M. Heal, M. L. Hansen, and K. M. Rickard, Maple V Learning Guide. Springer-Verlag, New York, 1998.

[16] R. M. Corless, Essential Maple, , Springer-Verlag, New York, 1995.

Appendix

Properties of Laplace Transform:

some of the important properties of Laplace transform which will be used in its applications are discussed below.

1. Definition of a Laplace Transform $F(s) = L[f(t)] = \int_0^\infty f(t)\ e^{-st} dt$ and $L^{-1}[F(s)] = f(t)$

2 Linearity: The Laplace transform of the sum, or difference, of two signals in time domain is equal to the sum, or difference, of the transforms of each signals, that is,

$$L\left[C_1 f(t) + C_2 g(t)\right] = C_1 L\left[f(t)\right] + C_2 L\left[g(t)\right]$$

3. Differentiation: If the function $f(t)$ is piecewise continuous so that it has continuous derivative $f^{n-1}(t)$ of order $n-1$ and a sectionally continuous derivative $f^{n}(t)$ in every finite interval $[0,\infty]$, then $L\left[f^{n}(t)\right] = s^{n} L\left[f(t)\right] - s^{n-1} f(0) - s^{n-2} f^{1}(0) - - - - - - - - f(0)$

3. Integration: $L\left[\int_0^t f(t) dt\right] = \frac{F(s)}{s}$

4. Laplace transform of Unit step signal $L\left[u(t-a)\right] = \frac{e^{-as}}{s}$

5. Second shifting theorem: $L\left[f(t-a)u(t-a)\right] = e^{-as} L\left[f(t)\right]$