



On one criterion for the permanent economy development

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ABSTRACT

A new method of investigation the equilibrium state of the economy is proposed. The classification of the equilibrium states is given. The basic principle of equilibrium the economy system to its environment is formulated. It is used to obtain the money circulation equation. On this basis the notion of the permanent economic development is defined. The analysis of the economy of four countries is given. The principle of adequacy of currency exchange and inflation rates to the factors that their determine is proposed.

Keywords: Exchange Rate, Forecast, Permanent Economy Development, Recession.

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1. Introduction

The purpose of the paper is to formulate the principle of the permanent economy development and to give the criterion of the closeness the economy system to the recession state. As it was shown one of the author in the papers [1, 2, 3, 4, 5] the closeness to the recession state is characterized by the quality of the economy equilibrium. If the degeneracy multiplicity of the economy equilibrium state is not high then we can say about the permanent economy development. In case if the degeneracy multiplicity is high then it means that the demand for the determining group of goods is low. With the sufficiently low demand for the determining group of goods, the economic system may fall into recession [1, 2, 3, 4, 5]. At the macroeconomic level, this is accompanied by a devaluation of the national currency, inflation, a fall in assets prices. The phenomenon was called the breakdown of the exchange mechanism. It is possible to reveal it in advance from the statistical information on the structure of production, outputs, foreign economic relations.

There are many factors destabilizing economy, however, all they are only a trigger to scrap the exchange mechanism. The hidden cause of the exchange mechanism breakdown is the quality of equilibrium state that can be determined by the structural characteristics of the economy [1, 2, 3, 4, 5]. How does monetary policy affect the recession in the economy? First, we study the influence of monetary policy on the exchange rate in nonlinear regression models and establish the equation of money circulation. The description of the national currency / dollar exchange rate by a random process satisfying the difference stochastic equation confirms that the logarithm of the exchange rate is a random walk without a drift. On this basis, the equation of money circulation in 2015 - 2017 was found. In this paper, based on the fact that the currency exchange rate of the country relative to the USA dollar is an indicator of the economy equilibrium with its environment we formulate

the canonical equation of money circulation which is a functional dependence between the quantity of money in the economy system, GDP and the refinance rate of the Central Bank of the state. Starting from this, we formulate the definition of the permanent economy development and its stability. The deviation from it means the different degree of approaching the economy equilibrium to the recession state. These notions we are applied to the investigation of the economy of Canada, China, Japan, Great Britain. After that we investigate the question of adequacy of the currency exchange rate to the factors that determine it. We formulate the dynamic stochastic model for the evolution of the currency exchange and inflation rates as unobservant random process the estimate of which we obtain by the Kalman method [6,7].

2. Exchange rate model.

Here we present the heuristic arguments for the equation managing the money circulation. Let us obtain the discrete evolution of the national currency / dollar exchange rate. Suppose that p_t and p_t^* are the logarithms of consumer price indices at the time moment t , then up to the constant depending only on the choice of the base year prices, $p_t - p_t^*$ is a logarithm of the the national currency / dollar exchange rate due to the monetary approach of the exchange rate formation. To obtain the equation managing the exchange rate, let us take the money circulation equations for two countries in the form

$$m_t = p_t + ky_t - \lambda r_t, \quad (1)$$

$$m_t^* = p_t^* + k^* y_t^* - \lambda^* r_t^*, \quad (2)$$

where m_t, y_t, r_t are the logarithms of the money supply, the gross domestic product and the refinance rate of the Central Bank at the moment t , and $k, \lambda, k^*, \lambda^*$ are some constants. Equation (2) refers to the foreign

economy. Using that the exchange rate

$$s_t = p_t - p_t^* + a, \tag{3}$$

and the formulae (1), (2), we have

$$s_t = a + m_t - m_t^* - (ky - k^* y^*) + \lambda i_t - \lambda^* i_t^* \tag{4}$$

Equation (4) requires many assumptions. Among them, the flexibility of prices in both economies. Arguments for neglect of the market for goods, labor, foreign economic relations, and bonds is difficult. We simplify the model assuming that $k = k^*$, $\lambda = \lambda^*$. From (4) we obtain

$$s_t = a + m_t - m_t^* - k(y - y^*) + \lambda(i_t - i_t^*). \tag{5}$$

The coefficients entering into equation (5) must be determined. Simple relation between the exchange rate, the consumer price indices, the gross domestic products, the money supply entering in money circulation equation (5) can not hold deterministically in view of the number of restrictions for the monetary model to be valid. Just the monetary circulation equation itself is not known.

Further, we assume that the deviation of the left side of (5) from the right one is a random process. Therefore, suppose that the discrete random process $\zeta_{k+1,i}$, $i = \overline{-p, 0}$, and k factors $X_i = \{x_{ji}\}_{j=0}^k$, $i = \overline{0, n}$, $X_0 = \{x_{j0}\}_{j=1}^k$, $x_{j0} = 1$, $j = \overline{1, k}$, are such that

$$\sum_{s=0}^p a_s \zeta_{k+1,i-s} - f_i(X_0, X_1, \dots, X_k) = \varepsilon_i, \quad i = \overline{1, n}, \tag{6}$$

where $a_0 = 1$, the random values $\zeta_{k+1,-p}, \dots, \zeta_{k+1,0}$ are known, the random values ε_i , $i = \overline{1, n}$, are independent with zero mean and dispersion

σ^2 , also, $\zeta_{k+1,-p}, \dots, \zeta_{k+1,0}$ do not depend on ε_i , $i = \overline{1, n}$, the functions $f_i(X_0, X_1, \dots, X_k)$, $i = \overline{1, n}$, are nonlinear functions of the factors. The most important case is when

$$f_i(X_0, X_1, \dots, X_k) = \sum_{j=0}^k b_j x_{ji}, \quad i = \overline{1, n},$$

and the equalities (6) become

$$\zeta_{k+1,i} - \sum_{j=0}^k b_j x_{ji} = \varepsilon_i, \quad i = \overline{1, n}. \tag{7}$$

Therefore, we further assume that the set of random variables $\zeta_{k+1,i}$, $i = \overline{1, n}$, are independent and have the joint normal distribution with mean

$$E\zeta_{k+1,i} = \sum_{j=0}^k b_j x_{ji} \text{ and dispersion } \sigma^2. \text{ It follows}$$

from (7) that the best forecast of a random process $\zeta_{k+1,i}$, $i = \overline{1, n}$, is its mean. To write the maximum likelihood estimate for vector $b = \{b_0, b_1, \dots, b_k\}$ and the determination coefficient which is the correlation of the process and its prediction in convenient way we introduce the matrix X , whose columns are the vectors X_i , $i = \overline{0, k}$, where

$$X_i = \{x_{ij}\}_{j=1}^n, \quad i = \overline{1, k}, \quad X_0 = \{e_j\}_{j=1}^n, \quad e_j = 1, \quad j = \overline{1, n}. \tag{8}$$

Then this matrix can be represented as

$$X = \begin{pmatrix} 1, x_{11}, \dots, x_{k1} \\ \dots \\ 1, x_{1n}, \dots, x_{kn} \end{pmatrix},$$

and the set of equations (7) as

$$\zeta_{k+1} = Xb + \varepsilon, \tag{9}$$

where $\zeta_{k+1} = \{\zeta_{k+1,i}\}_{i=1}^n$, $\varepsilon = \{\varepsilon_i\}_{i=1}^n$ are column vectors containing corresponding elements

$\zeta_{k+1,i}$ and ε_i . Let the matrix X rank be $k+1$.

Denote $A = X^T X$, it is symmetrical matrix having inverse one and the maximum likelihood estimate (MLE) for the coefficients of the column vector $b = \{b_0, b_1, \dots, b_k\}$ is unbiased and can be given in the form

$$\bar{b} = A^{-1} X^T \zeta_{k+1}. \quad (10)$$

The quality of the regression model is determined by the sample multiple correlation coefficient between the sample and the forecast

$$R_{n-1} = \frac{\sum_{i=1}^n (x_{k+1,i} - \bar{x}_{k+1})(x_{k+1,i}^1 - \bar{x}_{k+1}^1)}{\sqrt{\sum_{i=1}^n (x_{k+1,i} - \bar{x}_{k+1})^2 \sum_{i=1}^n (x_{k+1,i}^1 - \bar{x}_{k+1}^1)^2}}, \quad (11)$$

where $x_{k+1}^1 = X\bar{b} = \{x_{k+1,i}^1\}_{i=1}^n$, $\bar{x}_{k+1} = \frac{\sum_{i=1}^n x_{k+1,i}}{n}$,

$$\bar{x}_{k+1}^1 = \frac{\sum_{i=1}^n x_{k+1,i}^1}{n}.$$

It is easy to show that for R_{n-1}^2 the representation

$$R_{n-1}^2 = \frac{\sum_{i=1}^n (x_{k+1,i}^1 - \bar{x}_{k+1}^1)^2}{\sum_{i=1}^n (x_{k+1,i} - \bar{x}_{k+1})^2} \quad (12)$$

is valid. R_{n-1}^2 is called determination coefficient. The closer it is to unity, the better the quality of regression. For calculations, it is convenient to represent the determination coefficient in the form

$$R_{n-1}^2 = \frac{\langle x_{k+1} - \bar{x}_{k+1}, X_1 A_1^{-1} X_1^T (x_{k+1} - \bar{x}_{k+1}) \rangle}{\langle x_{k+1} - \bar{x}_{k+1}, x_{k+1} - \bar{x}_{k+1} \rangle} \quad (13)$$

where

$$X_1 = \begin{pmatrix} x_{11}^1, \dots, x_{k1}^1 \\ \dots \\ x_{1n}^1, \dots, x_{kn}^1 \end{pmatrix}, \quad (14)$$

$$x_{ji}^1 = x_{ji} - \frac{\sum_{i=1}^n x_{ji}}{n}, \quad A_1 = X_1^T X_1, \quad x_{k+1} - \bar{x}_{k+1} = \{x_{k+1,i} - \frac{\sum_{i=1}^n x_{k+1,i}}{n}\}_{i=1}^n, \text{ and } \langle a, b \rangle \text{ is scalar product of the}$$

vectors a and b . The estimation of maximum likelihood for dispersion σ^2 is

$$\sigma^2 = \frac{\langle x_{k+1} - \bar{x}_{k+1}, x_{k+1} - \bar{x}_{k+1} \rangle}{n}. \quad (15)$$

One can judge the quality of the factors on the basis of the Snedekor-Fisher statistics

$$\eta_{n-k-1, k+1} = \frac{\langle \varepsilon, (I - XA^{-1}X^T)\varepsilon \rangle}{\langle \varepsilon, XA^{-1}X^T\varepsilon \rangle} \frac{(k+1)}{(n-k-1)}, \quad (16)$$

where $\varepsilon = \{\varepsilon_i\}_{i=1}^n$, and the random values $\varepsilon_i = \frac{\zeta_{k+1,i} - E\zeta_{k+1,i}}{\sigma}$, $i = \overline{1, n}$, have the normal distribution $N(0,1)$ with zero mean and unit dispersion and are reciprocally independent. $E\zeta_{k+1,i}$ is mean value of the random value $\zeta_{k+1,i}$.

3. Canonical money circulation equation.

The equation of money circulation in the economy system can serve the additional instrument of the analysis of approaching of the economy system to the recession state. But there no exists the mathematical methods to obtain the functional dependence between the quantity of money in the economy system, gross domestic product (GDP) and the refinance rate of the Central bank. As it was shown by one of the author in the paper [1] under the conditions that the demand on a certain group of goods is equal zero then in the

economy system GDP decreases and the national currency devaluates. The depth of the decreasing of GDP depends on the quality of the equilibrium state, that is, from the degeneracy multiplicity of the economy equilibrium state.

The reaction of the Central bank on a such economic equilibrium state may be different: it decreases the refinance rate to increase the quantity of money in the economy system if the aim is that to reform the economy sector where the demand on a certain group of goods is sufficiently small, or it increases the refinance rate to decrease the quantity of money in the economy system to avoid the inflation of prices. All it depends on the assessment of the depth of recession in demand. We proceed from the assumption that the integral indicator of the equilibrium of the economy system with its environment is the exchange rate of the national currency relative to the USA dollar. Taking into account this fact and the monetary approach to the currency exchange rate formation we formulate the stochastic equation for the prognosis of the currency exchange rate relative to the USA dollar. The best prognosis of the currency exchange rate relative to the USA dollar is used to obtain the money circulation equation in the investigated period. This gives us the possibility to formulate the conditions of the permanent economy development. Using the statistics for four countries Canada, China, Japan, and Great Britain [8-17] for every of them the money circulation equation is established and on this basis the classification of the monetary policy of every of them is given. The economy system the money circulation equation of which satisfy the conditions of the permanent economy development is able to affect the quantity of money in the economy system changing the refinance rate. If the degeneracy multiplicity of the economy equilibrium state is not high then Central Bank is able to affect the quantity of money in the economy system.

In this section, using the statistics of 2015-2017

[8-17] for four contries Canada, China, Japan, and Great Britain, we establish a stochastic difference equation that national currency / dollar exchange rate satisfies. Since the quarterly statistical data 2015-2017 are known, in our case we use the following notation. The dependent variable $Y_i, i = \overline{1, n}$, describes national currency / dollar exchange rate evolution. Introduce the vector $Y = \{Y_i\}_{i=1}^n$. Let $X_0 = \{e_i\}_{i=1}^n, e_i = 1, i = \overline{1, n}$, be a unit vector-column and let $G_1 = \{G_i^1\}_{i=1}^n, G_2 = \{G_i^2\}_{i=1}^n$ be vectors-columns of gross domestic products for the investigated country and USA, respectively, and $M_1 = \{M_i^1\}_{i=1}^n, M_2 = \{M_i^2\}_{i=1}^n$ be money supply vectors for the investigated country and USA. Let $R_1 = \{R_i^1\}_{i=1}^n, R_2 = \{R_i^2\}_{i=1}^n$ be refinance rates of the Central Bank of the investigated country and the USA Federal Reserve System, correspondingly. Introduce the vectors

$$\begin{aligned} m_1 &= \log(M_1) = \{\log(M_i^1)\}_{i=1}^n = \{m_i^1\}_{i=1}^n, \\ m_2 &= \log(Y * M_2) = \{\log(Y_i M_i^2)\}_{i=1}^n = \{m_i^2\}_{i=1}^n, \\ r_1 &= \log(R_1) = \{\log(R_i^1)\}_{i=1}^n = \{r_i^1\}_{i=1}^n, \\ r_2 &= \log(R_2) = \{\log(R_i^2)\}_{i=1}^n = \{r_i^2\}_{i=1}^n, \\ g_1 &= \log(G_1) = \{\log(G_i^1)\}_{i=1}^n = \{g_i^1\}_{i=1}^n, \\ g_2 &= \log(Y * G_2) = \{\log(Y_i G_i^2)\}_{i=1}^n = \{g_i^2\}_{i=1}^n, \\ y &= \log(Y) = \{\log(Y_i)\}_{i=1}^n = \{y_i\}_{i=1}^n. \end{aligned}$$

Let the vector $\{y_i\}_{i=1}^n$ be a random process satisfying the stochastic equation

$$y_i = b_0 + b_1(m_i^1 - m_i^2) + b_2(g_i^1 - g_i^2) + b_3(r_i^1 - r_i^2) + \varepsilon_i, \quad i = \overline{1, n}, \quad (17)$$

where ε_i are independent random values distributed by the law $N(0, \sigma)$. Introduce the vector-columns $X_1 = m_1 - m_2, X_2 = g_1 - g_2, X_3 = r_1 - r_2$, where $X_i = \{x_{ij}\}_{j=1}^n, i = \overline{0, 3}$. We have the problem of the parameters estimation as in the problem (7) with $b_1 = 1$. Let b_0, b_2, b_3 be a

such parameters. From the equation (24) we obtain the best prognosis for the currency exchange rate

$$Y_i = e^{b_0} \frac{M_i^1}{M_i^2 Y_i} \frac{[G_i^1]^{b_2}}{[G_i^2 Y_i]^{b_2}} \frac{[R_i^1]^{b_3}}{[R_i^2]^{b_3}}, i = \overline{1, n}. \quad (18)$$

From the equation (18) we obtain the equality

$$M_i^1 = e^{-b_0} [Y_i]^2 M_i^2 [G_i^2 Y_i]^{b_2} [R_i^2]^{b_3} [G_i^1]^{-b_2} [R_i^1]^{-b_3}, i = \overline{1, n}. \quad (19)$$

Let us introduce the denotations

$$F_i = e^{-b_0} [Y_i]^2 M_i^2 [G_i^2 Y_i]^{b_2} [R_i^2]^{b_3}, f_i = \log F_i, i = \overline{1, n}$$

, and let us consider the problem

$$\min_{a_0, a_2, a_3} \sum_{i=1}^n (f_i - a_0 - a_2 g_i^1 - a_3 r_i^1)^2. \quad (20)$$

Then the equation (19) can be represented in the canonical form

$$M_i^1 = e^{a_0} [G_i^1]^{a_2 - b_2} [R_i^1]^{a_3 - b_3}, i = \overline{1, n}, \quad (21)$$

where a_0, a_2, a_3 is a solution of the problem (20).

We call the functional dependence (21) between the variables M_i^1, G_i^1, R_i^1 the canonical equation of money circulation in the i -th period, $i = \overline{1, n}$.

Definition 1. We say that an economy system in its period of functioning is in the state of the permanent economy development if the canonical equation of the money circulation satisfies the conditions: $L_2 = a_2 - b_2 > 0$,

$$L_3 = a_3 - b_3 < 0.$$

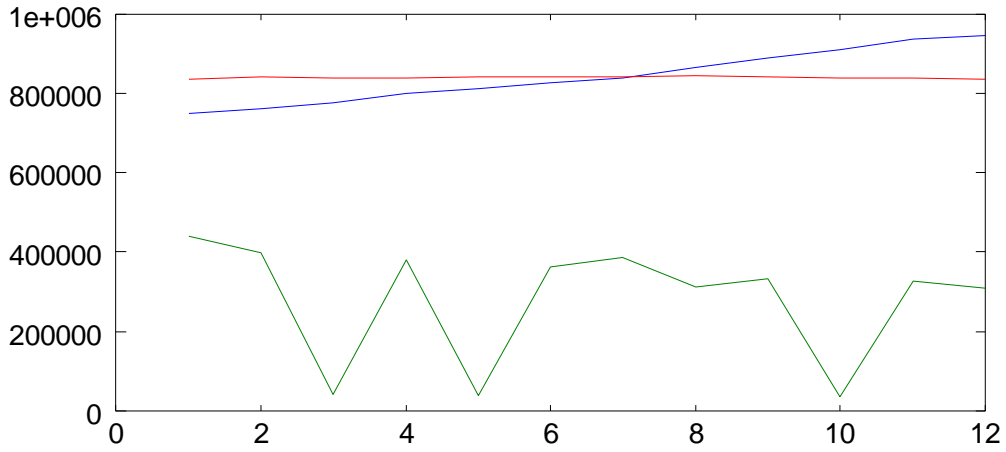
Below we give the analysis of the economy state for four countries on the basis of their statistics during 2015 -2017 years [8-17].

The canonical equation of money circulation for Canada does not correspond to the criterion of the permanent economy development since $L_2 = -7.0548 \cdot 10^{-4}$, $L_3 = -1.6444 \cdot 10^{-2}$ and the sign of L_2 is opposite to the sign in the Definition 1. Coefficient of determination of the problem (17) and standard deviation equal $R_{11} = 0.61449$, $\sigma = 0.062$, correspondingly. Really, during 2015 -2017 years [8-17] in the Canada economy the recession took place. During these years the Central Bank of Canada was decreasing the refinance rate trying to increase the demand on the produced goods in the economy. The quantity of money in the economy system was increased but these actions of the Central Bank did not affect the state of the economy for 2015 -2017 years [8-17]. If the velocity of money circulation V_1 , then

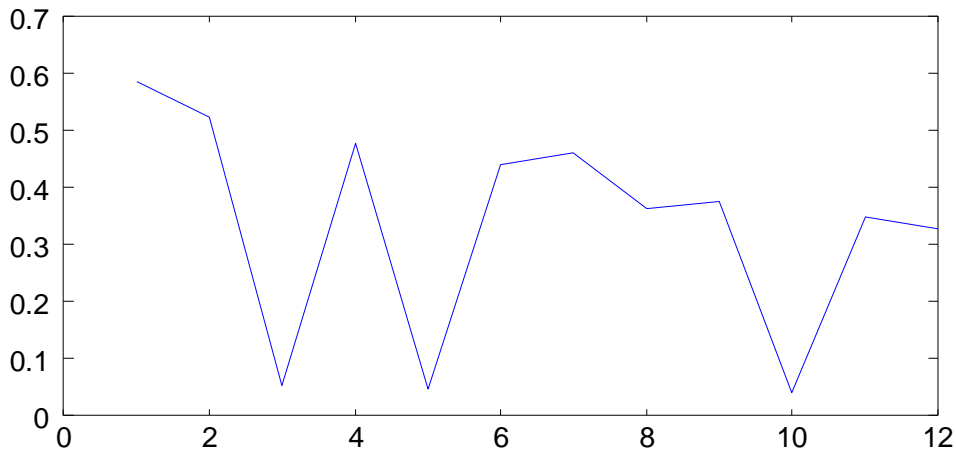
$$V_1 = \frac{G_1}{M_1}. \quad (22)$$

The velocity of money circulation had sharp swings for the Canada economy during 2015 -2017 years. Below we present the functional dependence of GDP, the money supply and the canonical money circulation equation in time for four countries during these years.

The canonical equation of money circulation for the China economy corresponds to the criterion of the permanent economy development since $L_2 = 1.82454$, $L_3 = -0.39902$, the coefficient of determination of the problem (17) and the standard deviation equal $R_{11} = 0.81716$, $\sigma = 0.078$, respectively. In the considered period for the China economy we observe the slow increasing of the GDP for the investigated period. The quantity of money in the economy of China was increasing in this period. The Central Bank of China was decreasing the refinance rate for the investigated period. We observe the adequate reaction of the Central Bank on the state of the economy for 2015 -2017 years.



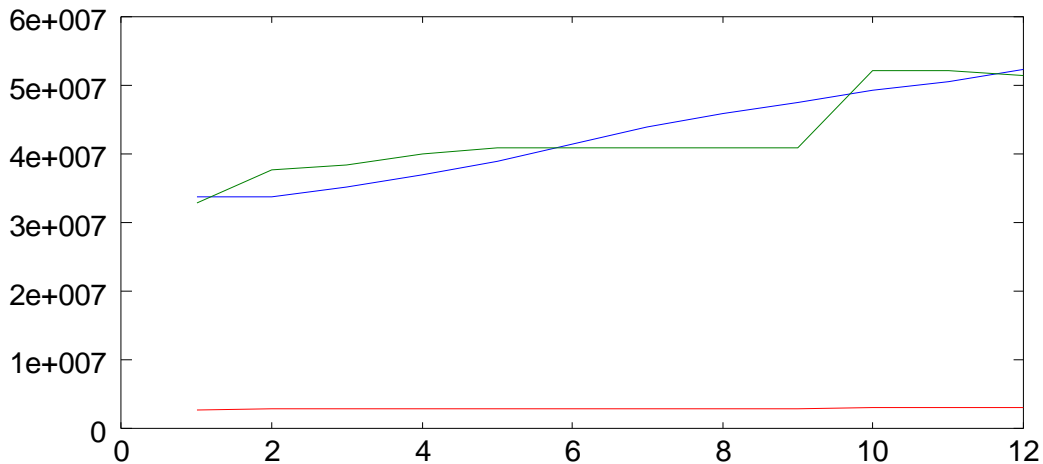
Canonical equation of money circulation for Canada economy during 2015 – 2017 years.



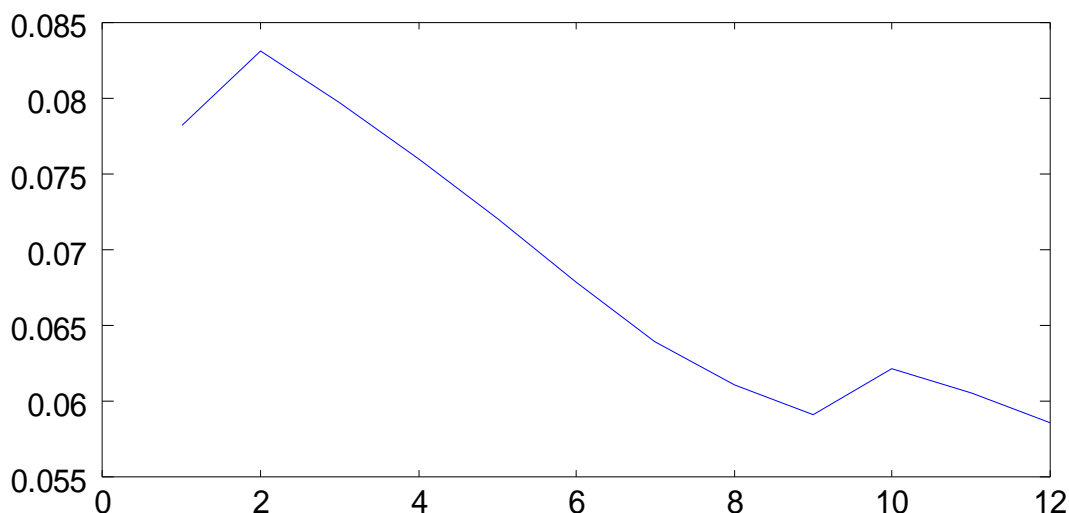
The velocity of money circulation for the Canada economy during 2015 – 2017 years.

At the beginning of the investigated period the velocity of money circulation was increasing and after that it began to decrease only at the end of the period it began to increase.

The functional dependence of the canonical money circulation for the China economy in time and velocity of money circulation is presented below.



Canonical money circulation equation for China economy during 2015 – 2017 years.



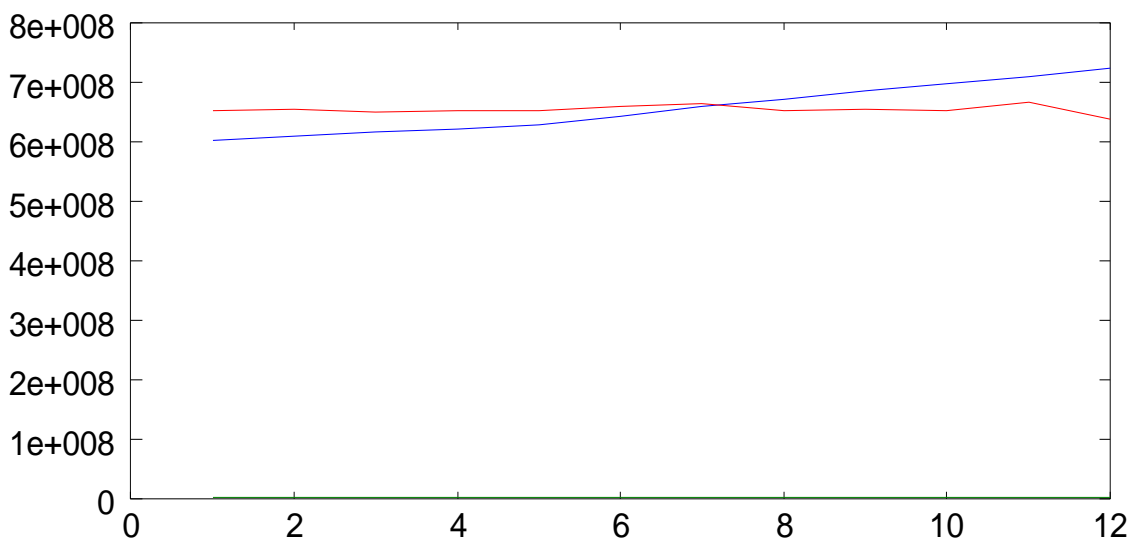
The velocity of money circulation for China economy during 2015 – 2017 years.

The canonical equation of money circulation for the Japan economy corresponds to the criterion of the permanent economy development with $L_2 = 0.130158$, $L_3 = -0.017477$, the coefficient of determination of the problem (17) and the standard deviation equal $R_{11} = 0.58143$, $\sigma = 0.058$, respectively. The values of L_2 , L_3 are very small. This indicates that the economy of Japan was close to the recession. At the beginning of the investigated period we observe that GDP was falling and after that it was oscillating. The quantity of money in the economy was increasing for all

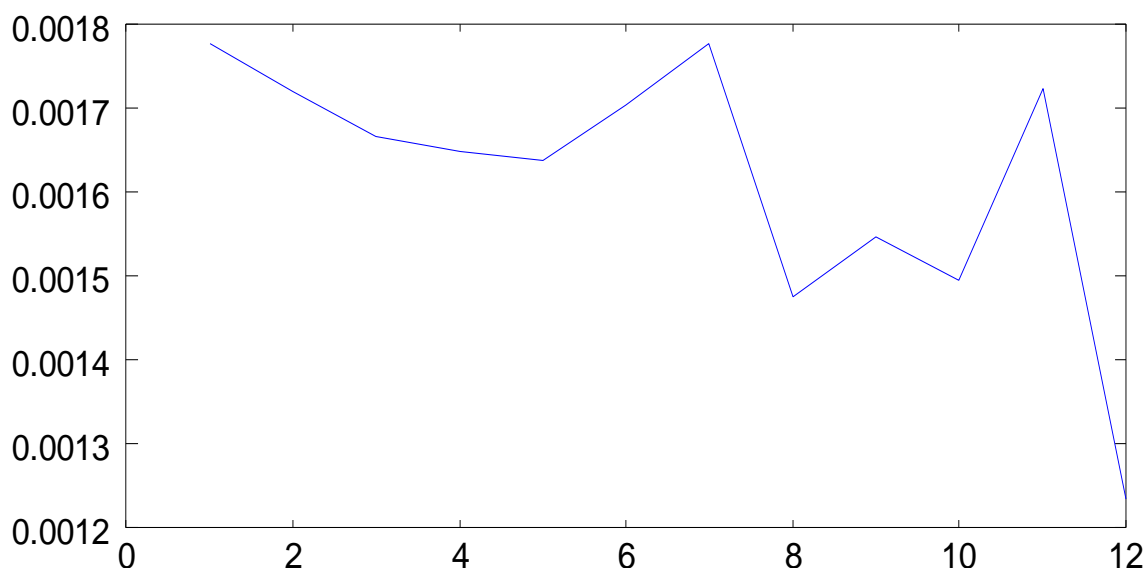
the time. At the beginning of the period the Central Bank of Japan began to decrease the refinancing rate and after that it began to increase it when the GDP began to increase.

At the beginning of the investigated period the velocity of money circulation was decreasing and after that it began to increase and then it began to oscillate with the sharp falling at the end of the period.

Below we present the functional dependence of GDP, the money supply and the canonical money circulation equation in time for the Japan economy for 2015 – 2017 of years.



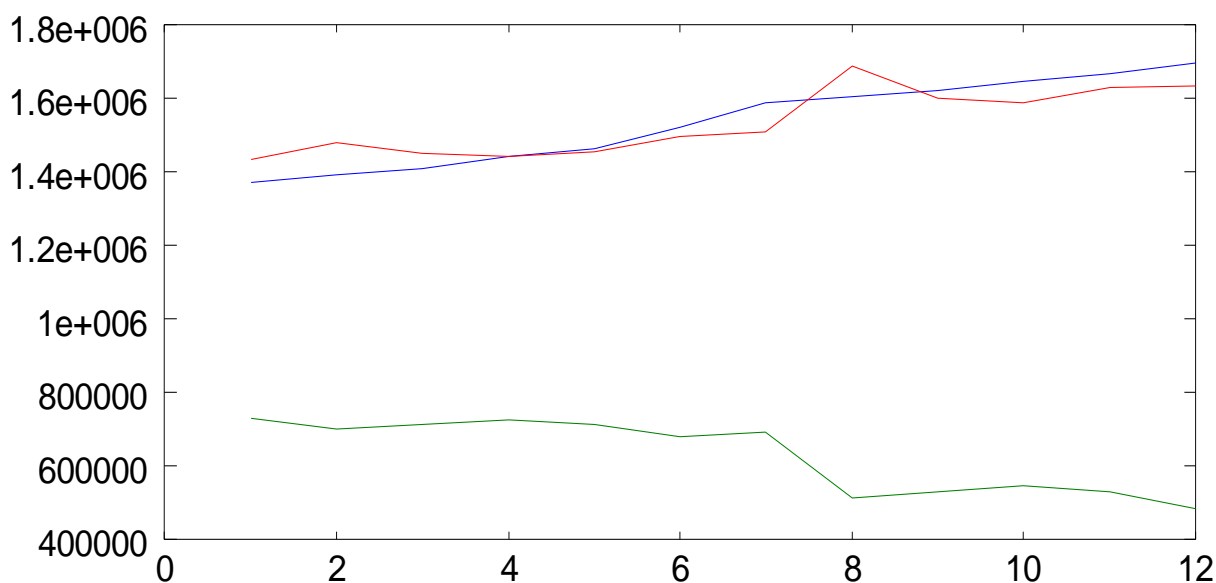
Canonical money circulation equation for the Japan economy during 2015 – 2017 years.



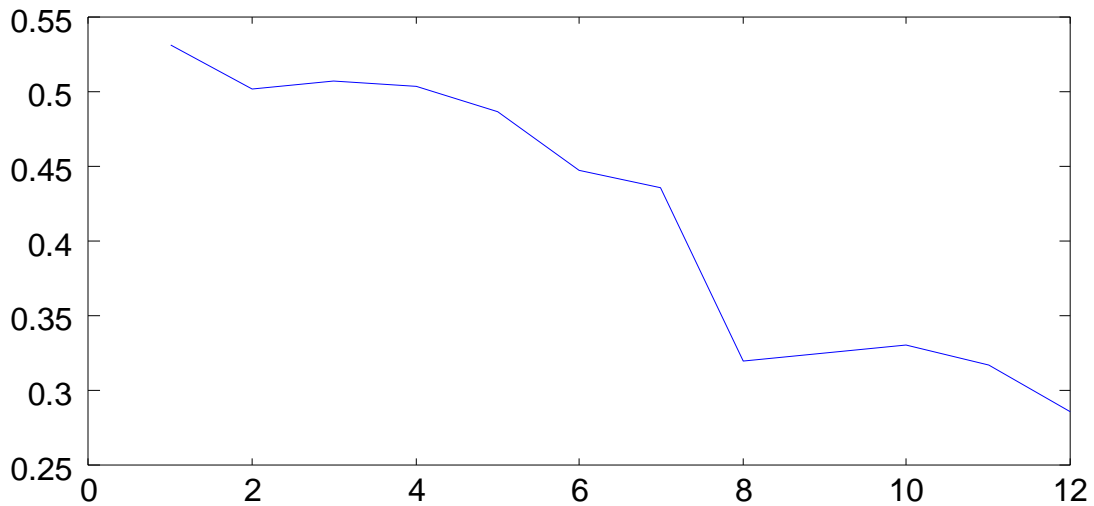
The velocity of money circulation for Japan economy during 2015 – 2017 years.

The canonical money circulation equation for the Great Britain economy does not correspond to the criterion of the permanent economy development with $L_2 = -0.16601$, $L_3 = -0.11516$, the coefficient of determination of the problem (17) and standard deviation equal $R_{11} = 0.86062$, $\sigma = 0.072$, respectively. The economy of Great Britain was in recession for all the period. At the beginning of the period the Central Bank of Great Britain

began to decrease the refinancing rate and after that it began to increase it and then the policy of Central Bank was changed to opposite one and at the end of the period it began again to increase the refinancing rate. All these actions of the Central Bank of Great Britain did not come to the stabilization of the Great Britain economy. The quantity of money in the economy was increasing for all the period. The velocity of money circulation was decreasing during all the time.



Canonical money circulation equation for Great Britain economy for 2015 – 2017 years.



The velocity of money circulation for Great Britain economy for 2015 – 2017 years.

4. Stability of the state of the economy equilibrium.

The obtained estimates of parameters in the equation (17) allow us to predict the exchange rate. To investigate the stability of the best prognosis let the random process y satisfy the stochastic equation

$$y_i = b_0 + b_1(m_i^1 - m_i^2) + b_2(g_i^1 - g_i^2) + b_3(r_i^1 - r_i^2) + b_4 y_{i-1} + \varepsilon_i, \quad i = \overline{1, n}, \quad (23)$$

where ε_i are independent random values distributed by the law $N(0, \sigma)$, and y_0 is known number.

From the equation (23) we obtain the best prognosis for the currency exchange rate

$$Y_i = e^{b_0} \frac{M_i^1}{M_i^2} \frac{[G_i^1]^{b_2}}{[G_i^2 Y_i]^{b_2}} \frac{[R_i^1]^{b_3}}{[R_i^2]^{b_3}} [Y_{i-1}]^{b_4}, \quad i = \overline{1, n}. \quad (24)$$

From the equation (24) we obtain the equality

$$M_i^1 = e^{-b_0} [Y_i]^2 M_i^2 [G_i^2 Y_i]^{b_2} [R_i^2]^{b_3} [Y_{i-1}]^{-b_4} [G_i^1]^{-b_2} [R_i^1]^{-b_3}, \quad i = \overline{1, n}. \quad (25)$$

Let us introduce the denotations $F_i = e^{-b_0} [Y_i]^2 M_i^2 [G_i^2 Y_i]^{b_2} [R_i^2]^{b_3} [Y_{i-1}]^{-b_4}$, $f_i = \log F_i$, $i = \overline{1, n}$.

And let us consider the problem

$$\min_{a_0, a_2, a_3} \sum_{i=1}^n (f_i - a_0 - a_2 g_i^1 - a_3 r_i^1)^2. \quad (26)$$

Then the equation (26) can be represented in the canonical form

$$M_i^1 = e^{a_0} [G_i^1]^{a_2 - b_2} [R_i^1]^{a_3 - b_3} \quad (27)$$

Definition 2. We say that an economy system in its period of functioning is in the stable state of the economy equilibrium if the canonical equation of the money circulation (27) satisfies the conditions: $L_2 = a_2 - b_2 > 0$, $L_3 = a_3 - b_3 < 0$.

As earlier, introduce the vectors-columns $X_1 = m_1 - m_2$, $X_2 = g_1 - g_2$, $X_3 = r_1 - r_2$, $X_4 = \{y_{i-1}\}_{i=1}^{n-1}$, where $X_i = \{x_{ij}\}_{j=1}^n, i = \overline{0, 4}$. We have the parameters estimate problem as in (14) with $b_1 = 1$. In this section we analyze the economy of four countries from the point of view how they were close to the recession state. We are interested in the question of stability of economy

development.

For the Canada economy the least squares estimation gives $L_2 = -5.96290 \cdot 10^{-4}$, $L_3 = -2.2361 \cdot 10^{-2}$. The determination coefficient $R_{11}^2 = 0.68490$ and the standard deviation estimate $\sigma = 0.056774$. From this we conclude that the Canada economy was in the unstable state of the economy equilibrium.

For the China economy the least squares estimation gives $L_2 = 0.91295$, $L_3 = -0.71165$. The determination coefficient $R_{11}^2 = 0.91295$, and the standard deviation estimate $\sigma = 0.053776$. From this we conclude that the China economy was in the stable state of economy equilibrium.

For the Japan economy the least squares estimation gives $L_2 = 0.204218$, $L_3 = 0.076257$. The determination coefficient $R_{11}^2 = 0.66589$, and the standard deviation estimate $\sigma = 0.052053$. From this we conclude that the Japan economy was in the unstable state of the economy equilibrium.

For the UK economy the least squares estimation gives $L_2 = -0.371970$, $L_3 = 0.013477$. The determination coefficient $R_{11}^2 = 0.88353$ and the standard deviation estimate $\sigma = 0.065746$. From this we conclude that the Great Britain economy was in the unstable state of the economy equilibrium.

5. Dependence of the exchange rate on the internal factors

We'll find out the effect of inflation, the budget deficit, the level of energy prices, current account balance, trade balance, and money supply on the exchange rate. As before, we assume the exchange rate to be a random process that satisfies the system of equations (6). More detailed, the exchange rate is influenced by the factors $X_i = \{x_{ij}\}_{j=1}^n, i = \overline{0,5}$, where $X_0 = \{e_i\}_{i=1}^n, e_i = 1, i = \overline{1,n}$, is an unit vector-column, $X_1 = \{x_{1j}\}_{j=1}^n$ is an inflation level, $X_2 = \{x_{2j}\}_{j=1}^n$ is a budget government deficit,

$X_3 = \{x_{3j}\}_{j=1}^n$ is an energy carriers' price, $X_4 = \{x_{4j}\}_{j=1}^n$ is a current account balance, $X_5 = \{x_{5j}\}_{j=1}^n$ is a trade balance, $X_6 = \{x_{6j}\}_{j=1}^n$ is a money supply. With $y = \log(Y) = \{\log(Y_i)\}_{i=1}^n = \{y_i\}_{i=1}^n$, we'll assume the random process y satisfy the set of equations

$$y_i = b_0 + b_1 x_{1i} + b_2 x_{2i} + b_3 x_{3i} + b_4 x_{4i} + b_5 x_{5i} + b_6 x_{6i} + b_7 y_{i-1} + \varepsilon_i, \quad i = \overline{1,n}, \quad (28)$$

where $\varepsilon_i, i = \overline{1,n}$, is the set of independent random values distributed by the normal law $N(0, \sigma)$ and y_0 is a fixed number. For the Canada economy maximum likelihood estimate for the vector $B = \{b_i\}_{i=0}^6$ gave the result $b_0 = -5.7 \cdot 10^{-1}$, $b_1 = -1.5 \cdot 10^{-2}$, $b_2 = -2.8 \cdot 10^{-11}$, $b_3 = 3.1 \cdot 10^{-3}$, $b_4 = -5.4 \cdot 10^{-12}$, $b_5 = -7.8 \cdot 10^{-1}$, $b_6 = 1.7 \cdot 10^{-12}$, $b_7 = -8.1 \cdot 10^{-2}$ for $y_0 = -0.150$. The determination coefficient $R_{11}^2 = 0.99$, and standard deviation estimate $\sigma = 4 \cdot 10^{-3}$.

For the China economy maximum likelihood estimate for the vector $B = \{b_i\}_{i=0}^6$ gave the result $b_0 = -1.6$, $b_1 = 1.1 \cdot 10^{-2}$, $b_2 = -2.2 \cdot 10^{-12}$, $b_3 = 9.2 \cdot 10^{-3}$, $b_4 = 5.5 \cdot 10^{-12}$, $b_5 = -9.5 \cdot 10^{-12}$, $b_6 = -2.6 \cdot 10^{-15}$, $b_7 = 2.3 \cdot 10^{-1}$ for $y_0 = -1.81$. The determination coefficient $R_{11}^2 = 0.95$, and the standard deviation estimate $\sigma = 1 \cdot 10^{-2}$.

The maximum likelihood estimate for the vector $B = \{b_i\}_{i=0}^6$ for the Japan economy gave the result $b_0 = 4.8$, $b_1 = -4.4 \cdot 10^{-2}$, $b_2 = 1.1 \cdot 10^{-9}$, $b_3 = -2.6 \cdot 10^{-1}$, $b_4 = 6.1 \cdot 10^{-10}$, $b_5 = 8 \cdot 10^{-9}$, $b_6 = 7.2 \cdot 10^{-15}$, $b_7 = 1.2 \cdot 10^{-1}$ for $y_0 = 4.74$. The determination coefficient $R_{11}^2 = 0.96$, and the standard deviation estimate $\sigma = 9 \cdot 10^{-3}$.

For the Great Britain economy maximum likelihood estimate for the vector $B = \{b_i\}_{i=0}^6$

gave the result $b_0 = 1.3$, $b_1 = -5.3 \cdot 10^{-2}$, $b_2 = 5.8 \cdot 10^{-13}$, $b_3 = -7.3 \cdot 10^{-4}$, $b_4 = -2.2 \cdot 10^{-12}$, $b_5 = -1.1 \cdot 10^{-11}$, $b_6 = 3.2 \cdot 10^{-12}$, $b_7 = 9.9 \cdot 10^{-1}$ for $y_0 = -0.457$. The determination coefficient $R_{11}^2 = 0.98$, and the standard deviation estimate $\sigma = 0.012$.

The latter indicates a very close relationship of the exchange rate with inflation, the budget government deficit, the level of energy prices, current account balance, trade balance, and money supply.

6. The Adequacy Of the Exchange And the Inflation Rates To Determining Factors.

In the previous section the question of the correspondence of the currency exchange to the factors that it define is investigated. Such relation as it was found is sufficiently close. We propose below a new method of the determination of the adequacy to the determining factors. Formation of the exchange and inflation rates in non-competitive economies may occur inadequately to the economically determining factors. This is due to the influence of monopoly on economic processes and the deformed taxation system, which contribute to inflationary processes and inadequate changes in the national currency / dollar rate. Using the introduced concept of adequacy of the exchange and inflation rates, we find out their relevance to the determining factors.

We believe that the national currency / dollar rate and the inflation rate are determined by the factors such as energy prices, budget government deficit, current balance of payments, trade balance, money supply.

The complexity of the problem lies in the fact that there is no unambiguous stochastic dynamic evolution of the economy. The main assumption is that the course of national currency / dollar and the rate of inflation are determined mainly by a certain set of factors and is an unobservable process whose estimate should be found.

Formulation of the problem. The evolution of factors $Y_i, i = \overline{1, N}$, is given that can affect the inflation and the exchange rates, as well as the evolution of the inflation rate and the exchange rate of national currency / dollar $x_i, i = \overline{1, N}$, is also given, where the dimensionality of the vector of factors is n , and the dimensionality of the vector x_i is equal to 2. One must clarify the adequacy of the influence of factors on the inflation and exchange rates. Below we assume that $N = 5$. Every vector $N_i, i = \overline{1, 5}$, has dimension 12. These are such vectors: budget deficit, the level of energy prices, current account balance, trade balance and money supply.

We'll solve the problem by constructing unobservable random process \bar{x}_k being adequate inflation rate and exchange rate within k -th period that evolves by the law

$$\bar{x}_{k+1} = F_{k,k+1} \bar{x}_k + w_k, \quad k = 0, N-1, \quad (29)$$

$$Y_{k+1} = H_k \bar{x}_k + v_k, \quad k = 0, N-1 \quad (30)$$

considering w_k and v_k to be Gauss white noises, i.e., $E w_k^i v_s^j = 0$, where

$$E w_k^i v_s^j = \delta_{ks} \delta_{ij} a^i, \quad a^i > 0, \quad i = 1, 2, \quad ,$$

$$E v_k^i v_s^j = \delta_{ks} \delta_{ij} b^i, \quad b^i > 0, \quad i = \overline{1, N},$$

with zero mean $E w_k = 0, E v_k = 0$. $F_{k,k+1} = E$ is unit 2×2 matrix.

We determine the matrix H_k from the condition of minimum for the functional

$$\min \sum_{i=1}^n \{ [y_i^k - h_{i1}^k x_1^k - h_{i2}^k x_2^k]^2 + [y_i^{k+1} - h_{i1}^k x_1^{k+1} - h_{i2}^k x_2^{k+1}]^2 \} , \quad (31)$$

where $H_k = [h_{i1}^k, h_{i2}^k]_{i=1}^n, Y_i^k = \{y_i^k\}_{i=1}^n, k = 1, N$.

At the beginning time moment

$$\hat{x}_0 = E \bar{x}_0 \tag{32}$$

$$P_0 = E (\bar{x}_0 - E \bar{x}_0)(\bar{x}_0 - E \bar{x}_0)^T. \tag{33}$$

We describe the evolution of the state estimate by the formula [6,7]

$$\hat{x}_k^- = F_{k,k-1} \hat{x}_{k-1}^- \tag{34}$$

Errors matrix within the time period $[k-1, k]$ is given by the expression

$$P_k^- = F_{k,k-1} P_{k-1}^- F_{k,k-1}^T + Q_{k-1} \tag{35}$$

Kalman matrix

$$G_k = P_k^- H_k^T [H_k P_k^- H_k^T + R_k]^{-1}$$

and

$$\hat{x}_k = \hat{x}_k^- + G_k (Y_{k+1} - H_k \hat{x}_k^-).$$

Updated bug matrix is

$$P_k = (E - G_k H_k) P_k^-.$$

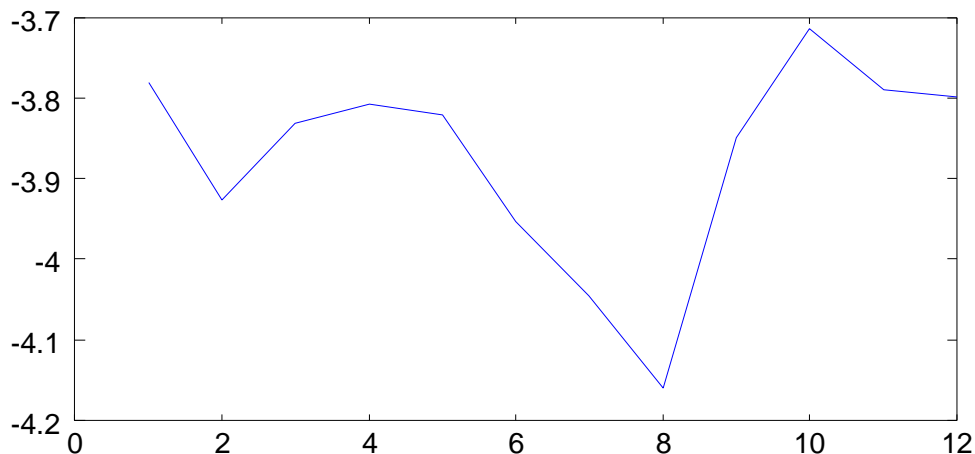
We suppose that $F_{k,k+1} = E$, where E is unit 2×2 matrix.

In the **Table1**, we show the quarterly change in the inflation rate and the Canada dollar / USD rate for 2015 – 2017 years.

Statistical data		Forecast	
1.93	0.86		
1.08	0.79		
0.9	0.81	-1.84	0.11
1.19	0.75	0.86	0.6
1.33	0.72	0.86	0.77
1.54	0.77	1.3	0.81
1.55	0.77	1.62	0.81
1.23	0.76	1.66	0.85
1.39	0.75	1.36	0.48
1.9	0.75	0.001	0.2
1.32	0.77	1.79	0.96
1.37	0.8	1.86	0.96

In correspondence with the prognosis of the currency exchange rate of Canada that is adequacy to the factors that determine it we observe the oscillation of one. This

corresponds to the oscillation of the current account balance of Canada presented on the plot below.



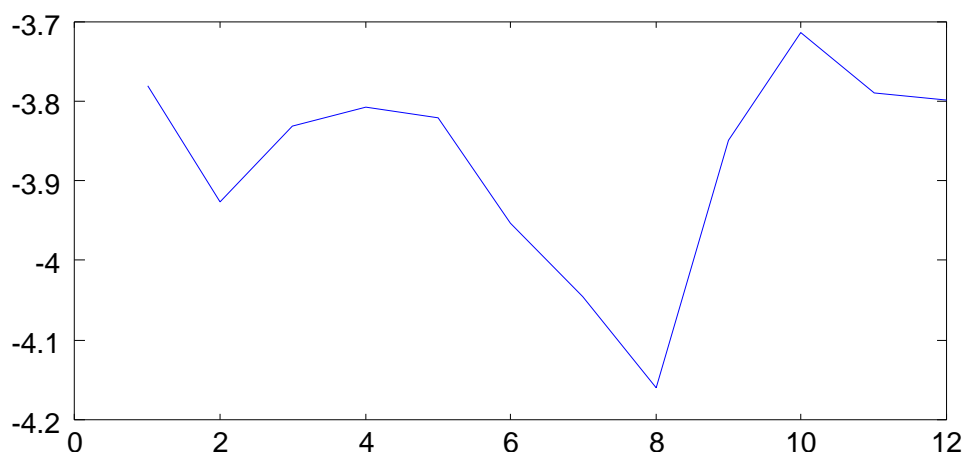
Current account balance of Canada for 2015- 2017 years.

In the **Table 2** we present the results of the China yuan / dollar rate. quarterly change in the inflation rate and the

Statistical data		Forecast	
1.5	0.163		
1.2	0.163		
1.37	0.163	1.27	0.164
1.73	0.157	1.44	0.164
1.47	0.154	1.28	0.165
2.13	0.155	1.66	0.156
2.07	0.150	2.14	0.155
1.67	0.150	2.12	0.154
2.17	0.144	3.45	0.152
1.4	0.145	2.16	0.159
1.4	0.147	1.25	0.143
1.6	0.150	1.4	0.146

We observe the change of the prognosis of the currency exchange rate of China that is adequacy to the factors that determine it. This

corresponds to the change of the current account balance of China presented on the plot below.

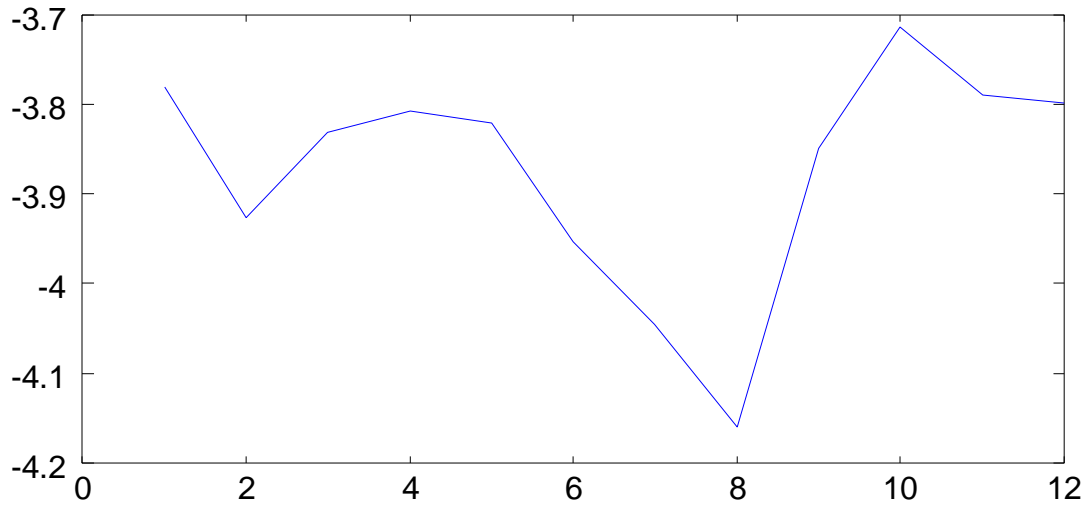


The current account balance of China for 2015 – 2017 years

In the **Table 3** we present the results of the quarterly change in the inflation rate and the

Japan yen / USD rate that corresponds to the current account balance presented below.

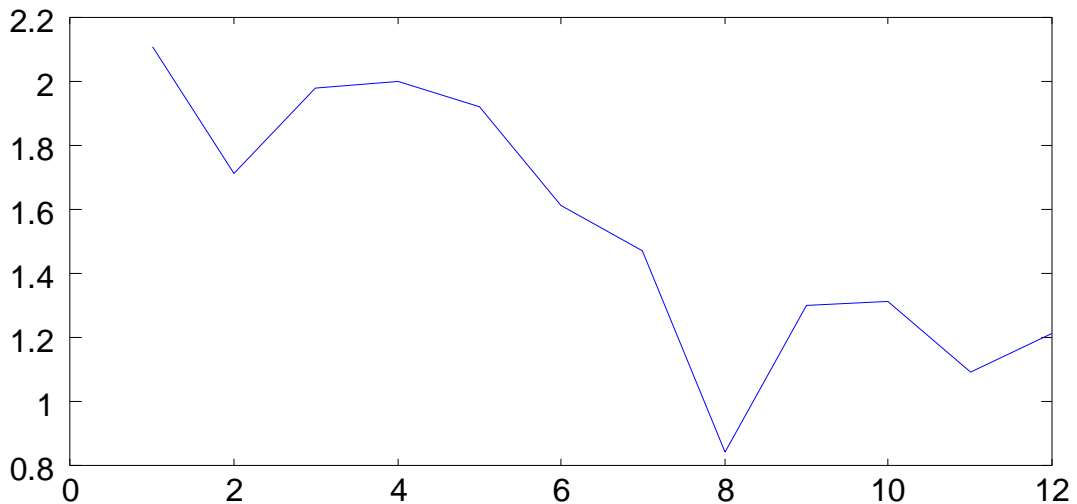
Statistical data		Forecast	
2.54	0.0083		
2.33	0.0083		
0.53	0.0082	0.31	0.0031
0.13	0.0083	2.16	0.0063
0.2	0.0083	0.51	0.0078
0.03	0.0089	0.15	0.0090
-0.33	0.0097	0.029	0.0094
-0.5	0.0099	1.35	0.0069
0.33	0.0086	0.023	0.0056
0.3	0.0089	-1.51	0.0044
0.37	0.0089	0.39	0.0100
0.6	0.0089	0.44	0.0111



In the **Table 4** we present the results of the Great Britain pound sterling /USD rate. quarterly change in the inflation rate and the

Statistical data		Forecast	
0.9	1.58		
0.1	1.58		
0.1	1.51	-0.24	1.53
0.1	1.55	0.1	1.56
0.1	1.52	0.09	1.48
0.3	1.43	0.1	1.45
0.4	1.43	0.35	1.45
0.7	1.31	0.01	1.15
1.2	1.24	0.37	1.13
2.1	1.24	1.38	1.24
2.7	1.28	1.57	1.23
2.8	1.31	2.2	1.14

We observe the change of the prognosis of the currency exchange of the Great Britain is adequacy to the factors that determine it. This corresponds to the change of the current account balance of Great Britain presented on the plot below.



The current account balance of Great Britain for 2015 – 2017 years

The needed statistical data relative to the economy state of four countries was taken from the resources [8-17].

7. Conclusion.

In the paper the principle of the permanent economy development is formulated. It is proved that the economy of China and Japan was in the state of the permanent economy development during 2015-2017. The Canada and Great Britain economy were in the recession. Investigation of the adequacy of currency exchange and inflation rates to the factors that their determine gives the positive results.

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