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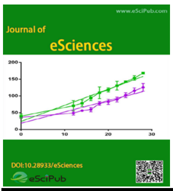

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Optimizing the Site Selection of Payment Stations Based on P-median Mathematical Model

Jiaying Yan[1]*, Gaozhen Ni[1]

[1]College of Electrical Engineering & New Energy, China Three Gorges University, Yichang, 443002, China.

ABSTRACT

Site selection is one of the classic problems in operations research. In this research, starting from a mathematical modeling example, a model was established to solve the site selection problem of 6 toll points in 52 cells. Taking the shortest distance between the residential area and the toll station as the objective function, this paper introduced 0-1 variables, established the P-median site selection model, and transformed the residents' payment query problem into a linear programming problem. The Lingo software was used to solve the problem of positioning the 6 points as 1, 6, 14, 26, 28 and 36. This paper successfully applies the site selection model to the site selection of toll station, which has certain reference value to the site selection in real life

Keywords: P-median Site Selection Model; Linear programming; Dijkstra algorithm

***Correspondence to Author:**
Jiaying Yan
College of Electrical Engineering & New Energy, China Three Gorges University, Yichang, 443002, China.

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1. Introduction

Site selection is one of the classic problems in operations research. It is widely used in production and life, logistics and even military affairs, such as site selection of factories, warehouses, emergency centers, fire stations, garbage treatment centers, logistics centers and missile warehouses. Site selection is one of the most important long-term decisions which will directly affect service mode, service quality, service efficiency and service cost, thus affecting profit and market competitiveness and even determining the fate of enterprises. Good site selection will bring convenience to people's life, reduce cost, expand profit and market share, improve service efficiency and competitiveness, while poor site selection will often bring great inconvenience and loss or even disaster. Therefore, the study of site selection has great economic, social and military significance.

In our life, water and electricity inquiry and toll points are an indispensable part of residents' life. If the site selection problem can be reasonably applied to the establishment of water and electricity toll points and the optimal allocation of water and electricity toll points around residents, it will bring great convenience to residents and improve the efficiency of national water and electricity management. Based on a mathematical modeling examples, this paper intends to find six toll points, making it most convenient for residents of 52 residential areas to inquire and pay water and electricity bills. Figure 1 below shows the road connections of the 52 residential areas in the title:

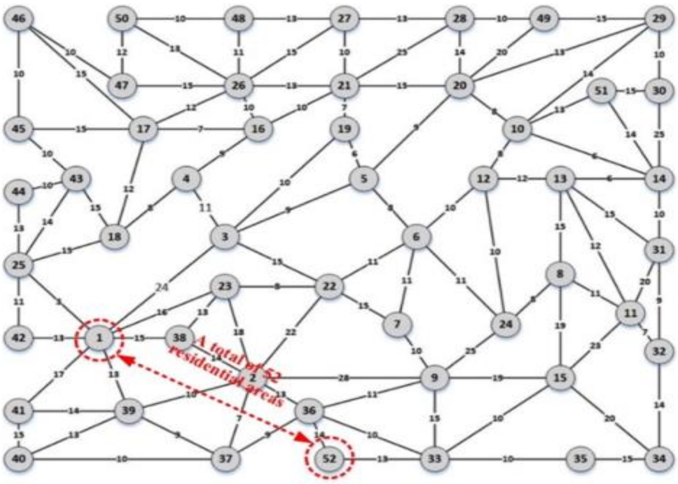

Fig.1：original title infographic—52 residential areas and road connections

2. Model hypothesis and symbol description

2.1 Model hypothesis

Hypothesis 1: the population of each residential area remains the same over a period of time.

Hypothesis 2: the distance between each residential area and the water and electricity toll point in this residential area is 0.

Hypothesis 3:residents go to the nearest water and electricity toll point to inquire and pay water and electricity bills.

2.2 symbol description

Symbol	Symbol description
N	A collection of 52 residential areas, including 6 water and electricity inquiries and toll points

i	Residential area of number i
j	The water and electricity toll point to be built of numbered j
H_i	The number of people in a numbered i residential area
d_{ij}	The shortest distance from a numbered i residential area to a numbered j residential area
w_{ij}	Represents the distance between two residential areas in the original weighted graph
P	Total number of water and electricity toll points
X_j	If in the residential area number j for the establishment of a toll station, $X_j=1$,otherwise 0
Y_{ij}	If the person from the residential area i comes to the toll point j for water and electricity charges, $Y_{ij}=1$,otherwise 0
G	The original weighted graph that was not dealt with in the problem
G'	A complete diagram showing the shortest distance between two regions
V	Vertex set of the original weighted graph

3. Problem analysis

Find six water and electricity toll points to make people in 52 residents living area conveniently query and pay bills, this is an optimal location model. Under the condition that the query station amount does not exceed 6, We need to reasonably allocate the six capture point location layout to minimize the sum of the weighted product between the distance of residential areas as well as toll points and population demand of each living area, Based on this, P-median Site Selection Model was established. Firstly, Dijkstra algorithm was used to obtain the minimum distance between 52 residential areas, and the P-media site selection model was solved to obtain the optimal toll point setting point and the residential area served. The flow chart of the whole problem solving process is shown in figure 2 below:

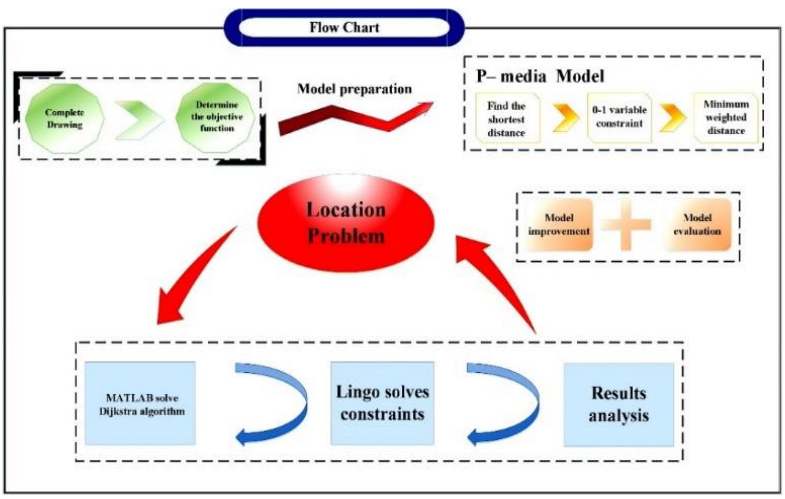

Fig.2 Flow chart of problem solving process

4. Data collection and analysis

In order to display the information in the original mathematical modeling example more intuitively, this paper draws the population of 52 residential areas and the number of road connections in each residential area as shown in the following figure 3:

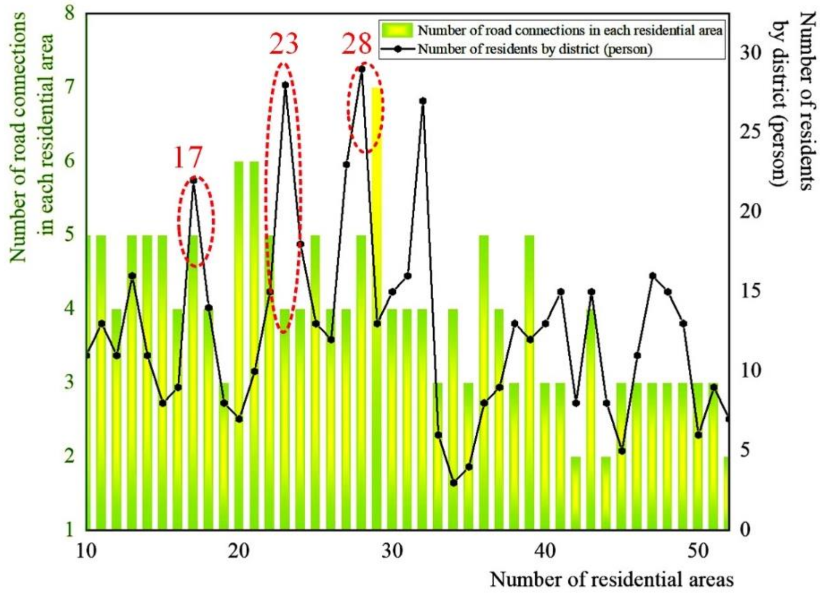

Fig 3: road connections and population of 52 residential areas

The number of local population and the number of roads connected to a certain area have a great impact on the water and electricity query and the construction site selection of the toll points. It is better to judge whether a certain residential area is suitable for the establishment of water and electricity toll points by multiplying the two quantities. As can be seen from the red area in the figure above, the three residential areas of area 17, 23 and 28 have the largest values, and it is speculated that water and electricity toll points will be established in some of these areas.

5. Establishment of P-median Site Selection Model

5.1 Preparation for Model Establishment

Considering that the road connection graph of the residential area given in the question is incomplete graph G, in order to make better use of the data given in the graph, the following algorithm is used to convert it into a complete graph G', which represents the shortest distance connecting roads of two residential areas.

5.1.1 Complete drawing

Dijkstra algorithm to find the shortest path between any two residential areas: the basic idea is to obtain the shortest circuit and distance of each vertex from u_0 to G in order of distance u_0 from near to far, until v_0, the end of the algorithm.

Step1:

let $l(u_0)=0$, for $v \neq u_0$, let $l(v)=\infty, S_0=\{u_0\}, i=0$

Step2: for each $v \in S_i (S_i=V \backslash S_i)$, use $\min_{u \in S_i}\{l(v), l(u)+w(uv)\}$ to replace $l(v)$, calculate $\min_{v\in S_i}\{l(v)\}$, call the vertex that gets to this minimum u_{i+1}, let $S_{i+1} = S_i \cap \{u_{i+1}\}$。

Step3: if $i = |V| - 1$, stop; if $i < |V| - 1$, use $i + 1$ to replace i, repeat *step2*, The result is the shortest distance between the 52 living areas d_{ij}, Get a complete picture G'。

5.1.2 Determine the objective function

Considering the situation that the payment sites are built on the road between two adjacent residential areas, in which case, the average distance from the toll points in each residential area is relatively large, the following models assume that all the 6 toll points are built in the residential area.

It is required to reasonably plan the locations of the six water and electricity toll points to facilitate residents to inquire and pay water fees. In other words, it is required to establish a weighted distance model to minimize the average distance $\bar{D}$ between all residents and the nearest water and electricity toll points. The model is as follows:

$$min\bar{D} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} H_i d_{ij} Y_{ij}}{\sum_{i=1}^{N} H_i}$$

where, H_i for the population of living area of numbers i、$\sum_{i=1}^{N} H_i$ is the total number of people for N residential area、d_{ij} is the shortest distance between the residential area and the toll point, Y_{ij} is a 0−1 variable.

5.1.3 determine the constraint conditions

(1) considering that each residential area can only inquire and pay fees at one water and electricity toll point, the variable Y_{ij} from residential area No.i to toll point No.j satisfies the following relationship:

$$Y_{ij} = \begin{cases} 1 \\ 0 \end{cases} \qquad i \in N, j \in N$$

$$\sum_{j=1}^{M} Y_{ij} = 1, \quad i \in \mathrm{N}$$

Among this,1 means that residents of No.i go to toll point of No.j while 0 means not, and the sum of Y_{ij} is 1.

(2) considering that the question requires the establishment of 6 toll points, whether the residential area No.j should be established as the variable X_j of toll points satisfies the following relationship:

$$X_j = \begin{cases} 1 \\ 0 \end{cases} \qquad j \in \mathrm{N}$$

$$\sum_{j=1}^{M} X_j = 6 \qquad i \in \mathrm{N}$$

Among this,1 means that a toll point shall be established in the residential area of No.j while 0 means not, and the sum of X_j is 6

(3) In consideration of the actual situation, the

residential area can only be served after the toll points are established. Therefore, there exists the following restrictive relationship between the two 0-1 variables: whether the residential area No.j is established as the variable X_j of the toll points, and whether residential area No.i comes to toll point No.j as the Variable Y_{ij}:

$$X_j \ge Y_{ij} \qquad i \in N, \; j \in N$$

5.2 determination of P-median Site Selection Model

To sum up, the P-media site selection model is established as follows:

$$min\bar{D} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} H_i d_{ij} Y_{ij}}{\sum_{i=1}^{N} H_i}$$

$$s.t.\begin{cases} \sum_{j=1}^{N} Y_{ij} = 1 \\ \sum_{j=1}^{N} X_j = 6 \\ X_j \ge Y_{ij} \\ X_j \in \{0,1\} \\ Y_{ij} \in \{0,1\} \\ i \in N, \; j \in N \end{cases}$$

6. Solution and result analysis of the model

6.1 solution of model

Using MATLAB software to solve Dijkstra algorithm, the minimum distance between each two residential areas of 52 was obtained as shown in table 1:

Table 1: the shortest distance between 52 living areas

	area1	area2	area3	area4	area49	area50	area51	area52
area 1	0	23	24	26	62	55	63	45
area 2	23	0	37	48	70	78	64	27
area 3	24	37	0	11	38	43	39	63
area 4	26	48	11	0	49	32	50	71
area 5	33	41	9	20	29	39	30	54
area 49	62	70	38	49	0	46	40	83
area 50	55	78	43	32	46	0	62	93
area 51	63	64	39	50	40	62	0	77
area 52	45	27	63	71	83	93	77	0

Combined with the shortest distance between any two residential areas solved above, the

Lingo software was used to solve the P-median site selection model to obtain the residential area number of toll points and the residential area number served by them, as shown in table 2 below.

Table 2: residential area number of water and electricity toll points

Toll point number	Service residential area number
1	18、23、25、38、39、41、42、43、44、45
6	3、5、7、8、12、19、22、24
14	10、11、13、29、30、31、32、34、51
26	4、16、17、21、46、47、48、50
28	20、27、49
36	2、9、15、33、35、37、40、52

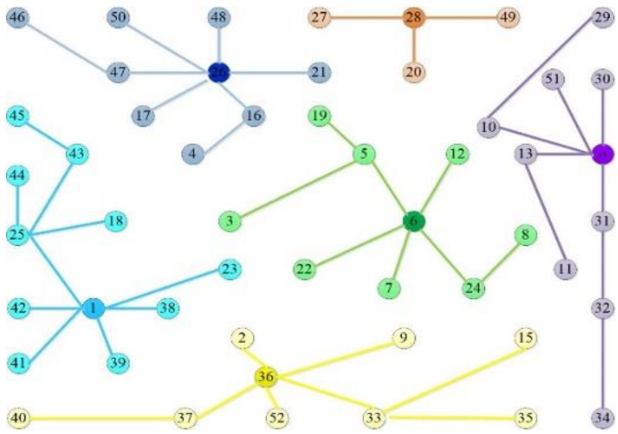

Fig. 4 water and electricity toll points and their service residential areas

As can be seen from the figure above, the 6 water and electricity inquiry and toll points are evenly distributed, covering roughly all the living areas of the whole county, ensuring the minimum average distance between all residents and their water and electricity toll points, which greatly facilitates residents to inquire and pay water and electricity bills.

7. Evaluation of the model

The P-median site selection model has been widely used in a variety of new energy supply station site selection problems. This model can express the problem in the form of integer linear programming, and solve the problem by using linear programming and integer programming, which makes use of the property of vertex optimization and reduces the search range of the target solution.

8. Model generalization and improvement

8.1 model improvement

For the optimal site selection problem, in addition to the shortest path, the equilibrium degree, construction cost, population and other aspects should also be considered. In addition, the cross-influences of multi-grouping results of multiple variables such as time can also be considered

8.2 model promotion

The P-median site selection model has certain practical significance. It has been widely used in the site selection of a variety of new energy supply stations, and can also be used in the dynamic planning of retail industry, warehouse location and charging stations for electric vehicles. In addition, emergency service facilities such as police stations, fire centers and medical aid centers can also be used.

9. Conclusion

Based on a mathematical model example, this paper established a P-median site model to solve the problem of setting up six toll point in 52 living area， the results respectively 1,6,14,26,28,36 makes the living area to the point of payment the shortest distance and also makes the payment point jurisdiction area equilibrium. It has important reference value for other site selection problems in real life, such as uniform site selection and high service efficiency.

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